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MULTIPLE HADRONIZATION AND THE MOMENTUM OF CHARMED PARTICLES: A SIGNAL OF THE QUARK-GLUON PLASMA

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ABSTRACT. The transverse momentum of charmed particles should be degraded by their passage through the mixed-phase fluid existing during the course of the quark-hadron phase transition. The mechanism is multiple hadronization, the repeated conversion of quark to meson and back as phase boundaries are crossed. The effect can be reduced by thermalization via ordinary collisions with particles of the plasma or the hadron gas, but passage through a homogeneous dense medium cannot duplicate the momentum degradation. The effect, if observed, thus constitutes a simple signal of the confinement phase transition in ultrarelativistic heavy ion collisions.

* Current address

In the continuing experimental effort to create a quark-gluon plasma in ultrarelativistic heavy ion collisions [1], a simple signal of plasma creation remains elusive. Tools suggested to date include the spectrum of continuum dileptons [2], the yield of resonances of the ψ family [3], the ratio of strange to non-strange particles [4], and the pattern of two-pion correlations [5]. All of these currently suffer from theoretical or practical difficulties. The first is heavily contaminated by backgrounds stemming from direct Drell-Yan emission and emission from the late hadron gas phase [2]; in the second, the effect of plasma suppression may be simulated by prosaic QCD effects and by scattering in the hadron gas [6]; the third is obscured by considerable processing (i.e., collision and recombination) in the course of the collision [7]; and the fourth mainly probes the structure of the collision region at the breakup stage, illuminating only weakly the intermediate stages of the process. At a more basic, conceptual level, the first three proposals are deficient in that they assume models for the quark-gluon plasma based (usually) on kinetic theory in a weakly interacting gas. Thus unforeseen complexity in the structure of the plasma, which is, after all, a non-linear, strongly interacting system [8], may vitiate the predictions. The fourth proposal, pion interferometry, is attractive in that it stands on the most basic features of hydrodynamic flow in the presence of a phase transition. Unfortunately, it cannot yet be tested because of insufficient statistics in the light-ion CERN experiments.

In this note, I propose the study of charmed particles as likely to yield a signal which is sensitive to features of the early evolution of a collision while relatively insensitive to the detailed structure of the produced matter. In particular, the transverse momentum distribution should be profoundly affected by the mere fact of a phase transition, irrespective of any but the grossest features of the high-temperature phase. Charmed quarks can only be produced in the initial hard collisions of partons present in the incoming nuclei (as long as the initial temperature is not above, say, 400 MeV [9]). Before they emerge, they must traverse the produced fluid, and they do so rather slowly, i.e., non-relativistically. Whatever the other characteristics of the high-temperature plasma, the charmed quarks are not confined; confinement only sets in when they hadronize in the course of the phase transition. The phase transition, however, is a complex kinetic process, which proceeds through the nucleation, growth, and coalescence of bubbles of the low-temperature phase—the true QCD vacuum, populated with a gas of hadrons. This "mixed phase" is very long-lived, with perhaps ten times the lifetime of the original plasma. During this long time, a charmed quark may emerge again and again into bubbles of confining vacuum, only to reenter the plasma each time (see Fig. 1). Each emergence requires hadronization. In each hadronization, the charmed quark loses energy; this process of *multiple hadronization* exerts a relatively large drag force on the quark, which will be reflected in the momentum of the charmed meson that is ultimately emitted. This drag thus offers a signal of the existence of a mixed phase in the course of the evolution of produced matter.

For a rough estimate of the drag force due to multiple hadronization, consider a charmed quark traveling through a region of mixed phase. On entering a bubble of vacuum, the charmed quark is forced to pull a light antiquark along to form a meson, losing energy in the process. (More precisely, the charmed quark stretches a QCD flux tube into the vacuum, and the tube breaks either by sucking in an existing antiquark or by creation of a light $q\bar{q}$ pair [10].) Upon reentering the plasma, the light antiquark is stripped off and diffuses away, forcing the heavy quark to find another partner in the next hadronization. We estimate the rate of energy loss (the drag force) via

$$\frac{dE}{dx} = \frac{\langle \Delta E \rangle}{\lambda}, \tag{1}$$

where $\langle \Delta E \rangle$ is the mean energy loss in a single passage through a bubble and λ is the mean free path for quark-bubble collisions. The latter is given by

$$\lambda^{-1} = \rho\sigma, \tag{2}$$

where ρ is the density of bubbles and σ the cross-sectional area of a bubble, assumed the same for all bubbles at a given time.

A lower bound on $\langle \Delta E \rangle$ comes from the kinematics of the $c \to D \to c'$ hadronization/ionization process. Suppose the $c \to D$ step occurs at its kinematic limit, with no energy loss to the plasma or to the production of other particles. Then we have $E_D = E_c$, and the heavy quark is slowed because of the greater mass of the meson. Suppose similarly that the heavy quark loses no more momentum in the $D \to c'$ step, i.e., that it keeps its velocity as the light quark is stripped off. Then $E'_c/m_c = E_D/m_D$, and thus

$$\begin{aligned} \Delta E \equiv E'_c - E_c &= -\left(1 - \frac{m_c}{m_D}\right) E_c \\ &\approx -\frac{1}{6} E_c. \end{aligned} \tag{3}$$

The minimal value for this energy loss is obtained by setting $E'_c = m_c \approx 1.5$ GeV, namely, $|\Delta E| > 300$ MeV. Thus even one extra hadronization causes substantial energy loss.

Creation of a D^* instead of a D, or fragmentation into $D\pi$, will more than double the energy loss. When the c quark is sufficiently energetic that threshholds are unimportant, these events are much more likely than D creation.

Returning to Eq. (2), we require estimates of ρ and σ. For these, we shall adopt a simple model of the phase transition kinetics. Begin with a plasma which cools to the transition temperature at proper time τ_p. (We work in the framework of the Bjorken scaling expansion [11], a one-dimensional expansion with fluid flow invariant under longitudinal boosts.) We suppose that at this instant there is enough fluctuation in the fluid to allow instant nucleation of bubbles of vacuum, with some density ρ_0; furthermore we assume that these bubbles can grow quickly enough to prevent any supercooling which would bring about additional nucleation at later times [12]. Thus the number of bubbles is conserved, and the one-dimensional scaling expansion gives

$$\rho = \rho_0 \frac{\tau_p}{\tau}. \tag{4}$$

Since all the bubbles grow in parallel, they present identical geometrical cross-sections to the charmed quark, and we shall take $\sigma = \pi r^2$ to be the effective cross section in (2). $r(\tau)$ may be calculated from

$$\frac{4}{3}\pi r^3 \rho = 1 - R, \tag{5}$$

where $R(\tau)$ is the volume fraction of plasma in the mixed phase, which drops from unity (at τ_p) to zero (when the plasma disappears). R in turn can be calculated via entropy conservation. Assuming that the entropy density of the plasma and of the hadron gas can be written as $s_p = c_p T_c^3$ and $s_h = c_h T_c^3$, respectively, and noting that the scaling expansion gives $s = R s_p + (1 - R) s_h = s_p \tau_p / \tau$, we obtain

$$1 - R = \frac{c_p}{c_p - c_h}\left(1 - \frac{\tau_p}{\tau}\right), \tag{6}$$

whence

$$\sigma\rho \propto (\tau - \tau_p)^{2/3}\tau^{-1}. \tag{7}$$

c_p and c_h are proportional to the effective number of degrees of freedom in the respective phases. If we take ideal gas thermodynamics of quarks and gluons on the one hand and pions on the other, we have $c_p/c_h = 37/3$.

This relation is only valid, of course, as long as the bubbles don't touch. After the bubbles begin to coalesce, the surfaces join in such a way as to lower the total surface area. In that case Eq. (7) provides an upper bound on the total area. Coalescence *must* occur by the point at which $1 - R = 0.74$, which is the packing fraction for a face-centered-cubic lattice of dense-packed bubbles (an admittedly fanciful possibility).

After appreciable coalescence has taken place, we have an inverted picture: Now there are shrinking droplets of plasma surrounded by the hadron gas. The passage of the charmed quark through the medium at this stage is however similar to that discussed above, namely, a series of hadronizations alternating with ionizations. We continue using Eqs. (1)–(2), where now ρ represents the density of droplets and σ the droplet cross section. Let us suppose that the number of droplets is roughly equal to the original number of bubbles. (Recall that these droplets are formed from the interstices of the lattice of bubbles.) Then we have as before $\rho = \rho_0 \tau_p/\tau$, while (5) is replaced by

$$\frac{4}{3}\pi r^3 \rho = R. \tag{8}$$

Using (6), we obtain

$$\sigma\rho \propto (\tau_h - \tau)^{2/3}\tau^{-1}, \tag{9}$$

where $\tau_h = c_p \tau_p / c_h$ is the time at which the phase transition ends, leaving a pure hadron gas.

The scaled variable $\rho\sigma/\rho_0^{1/3}$ is plotted against the scaled time τ/τ_p in Fig. 2, for both vacuum bubbles in the plasma and plasma droplets in the vacuum. Coalescence of bubbles will give an interpolation between the two curves. There is a period of peak drag lasting a time of perhaps $4\tau_p$ from the onset of the phase transition. Inserting the threshold value of $\Delta E = 300$ MeV, and guessing $\rho_0 = 1$ fm^{-3}, we obtain $|dE/dx| > 180$ MeV/fm during this period. Note that for an initial temperature of $T_0 = 1.5\,T_c$ at $\tau_0 = 1$ fm, the time scale $\tau_p = 3.4$ fm, so the period of peak drag is long indeed.

The bubble/droplet radius is shown in Fig. 3. The bubbles grow rapidly past a radius of 0.5 fm, which is large enough to accomodate the width of a flux tube and also its length (see below). The droplets created at coalescence stay above this size for a time period longer than the period of peak drag noted above, so here too the ionization/hadronization process can safely take place.

The effect of the drag force will be seen in the $p_\perp$ distribution of the final D mesons, but not in the longitudinal rapidity distribution. The reason is that the longitudinal motion is unaffected by the drag. The charmed quarks are created near $t = z = 0$ by the hard collisions of the incoming partons, and thus their trajectories are given by $z = v_z t$. This is also the equation governing fluid flow in the one-dimensional scaling expansion, and hence there is no initial relative motion between the charmed quarks and the fluid.

To discuss the $p_\perp$ distribution, let us begin with pp collisions, where the distribution of D mesons is measured to be [13]

$$\left(\frac{d\sigma}{dp_\perp^2}\right)_{pp\to DX} \propto e^{-p_\perp^2/p_{\perp 0}^2}, \tag{10}$$

with $p_{\perp 0} = 1.0$ GeV. We shall assume that this distribution is independent of longitudinal rapidity η. A D meson traveling with a typical $p_\perp$ of 1 GeV has a velocity of $0.5\,c$ in the frame where its $\eta = 0$, and $E = 2.06$ GeV in that frame. We take this to be indicative of the energy of the parent c quark, and also of the typical c quark created in nucleus–nucleus collisions. Such a quark thus has $p_\perp = 1.4$ GeV and a kinetic energy of 560 MeV. The collision region in a central Pb–Pb collision is some 15 fm in diameter, so it is clear that any quark created not too near the surface will stay within the collision region well into the mixed-phase stage, and furthermore will lose most of its initial momentum.

It is significant that this energy loss arises through interaction with phase interfaces, and not with constituent particles (quarks, gluons, or mesons). If the latter were the case, the drag force would be proportional to the momentum of the c quark, and the fluctuation-dissipation theorem would take effect [14]. The theorem states, simply, that a large drag force will be accompanied by rapid diffusion. The diffusion *in momentum space* would rapidly bring the charmed quark into thermal equilibrium. Then the observed $p_\perp$ distribution of D mesons would be thermal,

$$\left(\frac{d\sigma}{dp_\perp^2}\right)_{AA\to DX} \propto e^{-p_\perp^2/2m_D T}. \tag{11}$$

If the D were to escape during the mixed phase, and if we assume a transition temperature of 200 MeV, we would have $2m_D T = (775\text{ MeV})^2$, not very different from the coefficient in (10).

This points the way to a possible mechanism for avoiding the effect, namely, re-thermalization either by the plasma or by the hadron gas, either during the phase transition or after. The plasma drag coefficient γ_p was calculated perturbatively in [14], giving a result which may be approximated by

$$\gamma_p = c\,\frac{T^2}{m_c}, \tag{12}$$

with $c \simeq 0.3$. The corresponding time scale is given by $\tau_{\rm diff} \equiv \gamma_p^{-1}$. (The diffusion time scale is the same as the drag time scale by the fluctuation-dissipation theorem.) Using this result, perhaps without justification, at $T_c = 200$ MeV, we get an equilibration time of more than 20 fm, *without* taking into account the dilution of the plasma in the mixed phase. Of course, higher-order and non-perturbative effects could make this time much shorter when $T \sim T_c$; furthermore, the corresponding coefficient for a charmed meson moving through the hadron gas has not been calculated. It is thus not ruled out that re-thermalization may prevent degradation of the charmed meson momentum.

A question of perhaps greater importance, however, is whether scattering by a dense hadron gas (or other dense medium) may cause $p_\perp$ degradation similar to the multiple hadronization mechanism. Here the fluctuation-dissipation theorem indicates that the answer is no. As discussed above, large drag due to collisions with thermalized particles would be accompanied by rapid thermalization, hence yielding perhaps a 20% reduction in the mean $p_\perp$. Only drag due to bubbles, which move hydrodynamically rather than thermally, can give a dramatic momentum degradation.

Still, if T_c is much less than 200 MeV, we must entertain the possibility that the charmed particle re-thermalizes at a lower temperature, ending up with a low mean $p_\perp$. Likewise, if the charmed particle stays in the hadron gas after the transition concludes, will it cool with the medium? I believe both concerns can be dismissed. The plasma drag coefficient (13), even if it is somewhat larger because of strong-coupling effects, will decrease as T^2, where the exponent comes from a combination of the density of massless quanta ($\propto T^3$), their mean momentum ($\propto T$), and the Debye-screened scattering cross section ($\propto T^{-2}$). Thus the diffusion time scale will increase as T^{-2}. The drag coefficient in the hadron gas will drop as a larger power (because of the absence of Debye screening, and because of PCAC) or even exponentially (as the pion mass begins suppressing the equilibrium density of pions). Thus the charmed particle will decouple rapidly as the system cools, preserving in its momentum a relic of earlier times.

Finally, it is instructive to note the contrast between the multiple hadronization process and the perturbation of light-quark jets by the plasma [15]. The essential difference is the amount of time needed to break the flux tube into hadrons in the two cases. In common flux-tube hadronization pictures, such as the Lund scheme [16], hadronization typically occurs at a proper time $\tau_0 \sim 1$ fm. Thus the length of the flux tube at the time the leading hadron is formed is $z = \beta\gamma\tau_0 = (p/m)\tau_0$. For the slow-moving D meson discussed above, $z < 0.7$ fm, and hadronization can easily be completed within the confines of a bubble in the mixed phase. For a high-energy jet, on the other hand, z can be several hundred fm, leading to hadronization well outside the collision region and allowing a perturbative treatment of scattering of the leading quark by the plasma.

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REFERENCES

[1] H. Satz, H. Specht, and R. Stock, eds., *Quark Matter,* Proceedings of the 6th International Conference on Ultra-Relativistic Nucleus–Nucleus Collisions, Nordkirchen, FRG, August 1987, Z. Phys. **18** (1988) Nos. 1/2.
[2] K. Kajantie, M. Kataja, L. McLerran, and P. V. Ruuskanen, Phys. Rev. D**34** (1986) 811 and references therein.
[3] T. Matsui and H. Satz, Phys. Lett. **178B** (1986) 416.
[4] J. Rafelski and B. Müller, Phys. Rev. Lett. **48** (1982) 1066.
[5] G. Bertsch, M. Gong, and M. Tohyama, to appear in Phys. Rev. C;
P. V. Ruuskanen and D. Seibert, Phys. Lett. **213B** (1988) 227.
[6] J.-P. Blaizot, talk given at the 7th International Conference on Ultra-Relativistic Nucleus–Nucleus Collisions, Lenox, MAss., USA, Sept. 1988, to appear in proceedings in Nucl. Phys. A.
[7] J. Kapusta and A. Mekjian, Phys. Rev. D**33** (1986) 1304;
T. Matsui, B. Svetitsky, and L. McLerran, Phys. Rev. D**34** (1986) 2047;
P. Koch, B. Müller, and J. Rafelski, Phys. Rep. **142** (1986) 167.
[8] D. Gross, R. Pisarski, and L. Yaffe, Rev. Mod. Phys. **53** (1981) 43;
A. Hosoya and K. Kajantie, Nucl. Phys. **B250** (1985) 666;
J. Polonyi, Nucl. Phys. **A461** (1987) 279c.
[9] A. Shor, preprint (unpublished, 1988).
[10] B. Banerjee, N. Glendenning, and T. Matsui, Phys. Lett. **127B** (1983) 453.
[11] J. D. Bjorken, Phys. Rev. D**27** (1983) 140.
[12] K. Kajantie and H. Kurki-Suonio, Phys. Rev. D**34** (1986) 1719.
[13] M. Aguilar-Benitez *et al.,* Phys. Lett. **189B** (1987) 476.
[14] B. Svetitsky, Phys. Rev. D**37** (1988) 2484.
[15] D. A. Appel, Phys. Rev. D**33** (1986) 717;
J.-P. Blaizot and L. McLerran, Phys. Rev. D**34** (1986) 2739.
[16] B. Andersson, G. Gustafson, G. Ingelman, and T. Sjostrand, Phys. Rep. **97** (1983) 31.

FIGURE CAPTIONS

FIG. 1. (a) Passage of a charmed quark through the mixed phase early in the phase transition. The c quark enters a bubble of vacuum and hadronizes into a D meson, which then reenters the plasma and dissociates, leaving quark c' with lower energy than c. (b) Later in the phase transition, we have a D meson which enters a droplet of plasma and dissociates. The resulting c quark then reemerges into the vacuum and hadronizes into meson D'.

FIG. 2. Inverse mean free path for c quark in the mixed phase, scaled by the density of bubbles at nucleation. The solid curve corresponds to scattering by bubbles, the endpoint due to bubbles at their maximum packing fraction. The dashed curve corresponds to scattering by droplets thereafter.

FIG. 3. Radius of bubbles before coalescence (solid curve) and of droplets afterwards (dashed curve).