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PAIR PRODUCTION IN A STRONG ELECTRIC FIELD

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ABSTRACT. We investigate the mechanism of pair creation in scalar QED from spatially homogeneous strong electric fields in 1+1 dimensions. Solution of the semiclassical field equations shows particle creation followed by plasma oscillations. We compare our results with a model based on a relativistic Boltzmann–Vlasov equation with a pair-creation source term related to the Schwinger mechanism. The time evolution of the electric field and the current obtained from the Boltzmann–Vlasov model is surprisingly similar to that found in the semiclassical calculation.

The problem of spontaneous pair production in the presence of an external electric field has been investigated by many authors [1–6]. The most commonly used formula for the (boson) pair creation rate is based on Schwinger's one-loop calculation [2] for a constant and homogeneous electric field E,

$$w = \frac{m^4}{(2\pi)^3}\left(\frac{E}{E_0}\right)^2 \sum_{n=1}^{\infty} \frac{1}{n^2} \exp(-n\pi E_0/E) \quad , \quad E_0 = m^2/e \tag{1}$$

(in units such that $\hbar = c = 1$). In this expression the mutual interactions of the particles produced and the effect of these particles on the original electric field (back reaction) are not taken into account.

This formula has been used in modeling particle production in the central rapidity region in high-energy nucleus–nucleus collisions [7–9]. The production process is viewed as the tunneling of quark-antiquark and gluon pairs in the presence of a background color-electric field, formed after the collision in the region between the two nuclei that have been color-charged by the exchange of soft gluons. One usually studies only an Abelian version of the theory; equation (1) is used as if the electric field were constant, and the time-dependence of E is put in by hand.

A scheme for solving the quantum back-reaction problem in scalar QED has been offered by Cooper and Mottola [10]. We present here the results of numerical analysis based on this formalism. One would expect substantial pair production if the initial electric field is strong enough (of order E_0), and at large times one should see plasma oscillations. Studying this requires a numerical calculation that proves quite challenging. It is useful to investigate this problem in 1+1 dimensions as a first step in testing numerical procedures since the requirements of renormalization are relatively trivial in that case [11].

We treat the electric field as a semiclassical or mean field, which is justified if the electric field is strong enough, or if we take the matter field to be a scalar field in the large-N limit [10]. With a view towards implementing an adiabatic renormalization scheme, we restrict ourselves to a spatially uniform field. The full scheme is needed when one solves the (3+1)-dimensional problem, but infinities must be subtracted even in the (1+1)-dimensional case.

We also assume a special, albeit natural, initial configuration of the charged-matter field—the adiabatic vacuum. This corresponds to the initial configuration assumed by Schwinger [2] which results in (1) for a constant electric field.

The scalar QED coupled equations of motion in the mean field approximation (and without the ϕ^4 self-interaction term of Ref. 10) consist of the Klein-Gordon equation,

$$[(\partial^\mu + ieA^\mu)(\partial_\mu + ieA_\mu) + m^2]\Phi(x) = 0 \quad , \tag{2}$$

and the semiclassical Maxwell equations,

$$\partial_\mu F^{\mu\nu} = \langle 0|j^\nu|0\rangle \quad , \tag{3}$$

where $|0\rangle$ is the adiabatic vacuum and j^ν is the current of the charged scalar field. Spatial homogeneity causes the charge density j^0 to vanish everywhere. The Maxwell equations in the gauge $A_0 = 0$ consist of the single equation

$$\ddot{A} = \langle 0|j|0\rangle \quad , \tag{4}$$

where $A \equiv A_1$ and $j \equiv j_1$. Homogeneity also enables us to expand the boson field operator Φ in terms of plane waves, defining the operators a_k and $b_k^\dagger$ via

$$\Phi(x,t) = \int \frac{dk}{2\pi} \left[f_k(t)a_k + f_k^*(t)b_{-k}^\dagger\right] e^{ikx} \quad , \tag{5}$$

where to satisfy (2) we demand

$$\frac{d^2 f_k(t)}{dt^2} + \omega_k^2(t) f_k(t) = 0 \quad , \quad \omega_k(t)^2 \equiv [k - eA(t)]^2 + m^2 \quad . \tag{6}$$

The canonical commutation relations imply that the $f_k(t)$ can be expressed in terms of the real and positive functions $\Omega_k(t)$,

$$f_k(t) = \frac{1}{\sqrt{2\Omega_k(t)}} e^{-i\int^t \Omega_k(t')dt'} \quad , \tag{7}$$

where the $\Omega_k(t)$ obey the equation of motion

$$\Omega_k^2(t) = -\ddot{\Omega}_k/(2\Omega_k) + \frac{3}{4}\left(\dot{\Omega}_k/\Omega_k\right)^2 + \omega_k^2(t) \quad . \tag{8}$$

Upon subtracting the divergent part of the current, Eq. (4) becomes

$$\ddot{A} = \langle 0|j|0\rangle = e\int \frac{dk}{2\pi}(k - eA)\left[\frac{1}{\Omega_k(t)} - \frac{1}{\omega_k(t)}\right] \quad . \tag{9}$$

Eqs. (8) and (9) define the numerical problem.

We show in Fig. 1 the time evolution of the scaled electric field $\tilde{E} \equiv eE/m^2$ and induced current $\tilde{j} \equiv ej/m^3$, as functions of $\tau \equiv mt$. Stability was attained for these results, as well as for those presented below, with a time step $d\tau = 10^{-4}$ and a momentum grid with $d\tilde{k} \equiv dk/m = 0.004$, forcing very long running times. With strong initial electric fields we find that the induced current increases rapidly and becomes saturated at a constant value for some time, after which clear plasma oscillations are seen.

The saturation of the first oscillation is easy to understand. In a classical kinetic picture, we have $j = 2en\langle v\rangle$, where n is the density of particles (or antiparticles) and v is their velocity. j saturates as v is driven to the speed of light by the strong electric field. In subsequent oscillations, n is larger and E is weaker, so the particles remain nonrelativistic even at the peak of the current. This analysis gives us a method for estimating the number of pairs created in the first rise of j, allowing us to evade the ambiguities in the definition of particle number which are inherent in the adiabatic approach. We display a graph of the peak current in the first oscillation, as a function of the initial field, in Fig. 2.

As time progresses, there develops a highly oscillatory behavior (as a function of k) for the integrand in Eq. (9). We see this behavior also in the expectation value of the number operator for large times

$$\begin{aligned} \frac{1}{V}\frac{dN}{dk} &= \frac{1}{2\pi}\left\langle 0\left|a_k^{\mathrm{out}\dagger}a_k^{\mathrm{out}}\right|0\right\rangle \\ &= \lim_{t\to\infty}\frac{1}{8\pi\Omega_k\omega_k}\left[(\Omega_k - \omega_k)^2 + \frac{1}{4}\left((\dot{\Omega}_k/\Omega_k) - (\dot{\omega}_k/\omega_k)\right)^2\right] \quad , \end{aligned} \tag{10}$$

shown in Fig. 3. These oscillations are the result of oscillations in *time* with the momentum-dependent frequency $\Omega_k(t)$, as seen in (7). They disappoint the hope of a simple power-law fall off at large k, and hint at some likely difficulties in the implementation of the adiabatic renormalization scheme in 3+1 dimensions.

Many calculations in the realm of nucleus–nucleus collisions have been based on a phenomenological model employing a relativistic Boltzmann-Vlasov equation with a source term to describe particle creation [12–14]. In order to gauge the validity of this model, let us compare its results with those of our semiclassical analysis. The relativistic kinetic equation in the presence of a homogeneous electric field is

$$\frac{\partial f}{\partial t} + eE\frac{\partial f}{\partial p} = \frac{dN}{dt\,dx\,dp} = \frac{|eE(t)|}{2\pi}\log\left[1+\exp\left(-\frac{\pi m^2}{|eE(t)|}\right)\right]\delta(p) \quad , \tag{11}$$

where $f(p,t)$ is the (x-independent) classical phase space distribution and the right-hand side is the boson pair-production rate in $1+1$ dimensions. (We assume that the particles are produced at rest, i.e., the source term is proportional to $\delta(p)$.) The solution of Eq. (11) is given by

$$\begin{aligned} f(p,t) &= \int_0^t dt'\,\frac{|eE(t')|}{2\pi}\log\left[1+\exp\left(-\frac{\pi m^2}{|eE(t')|}\right)\right]\delta(p-eA(t')+eA(t)) \\ &= \frac{1}{2\pi}\sum_i \log\left[1+\exp\left(-\frac{\pi m^2}{|eE(t_i)|}\right)\right] \quad , \end{aligned} \tag{12}$$

where the t_i's fulfill $p+eA(t)-eA(t_i)=0$. The field equation for A is

$$\frac{d^2A}{dt^2} = j_{total} = j_{cond} + j_{pol} \tag{13}$$

where the conduction current is

$$j_{cond} = 2e\int\frac{dp}{\epsilon_p}\,p\,f(p,t) \quad , \tag{14}$$

with $\epsilon_p \equiv \sqrt{p^2+m^2}$, and the polarization current is [13]

$$j_{pol} = \frac{2}{E}\int dp\,\epsilon_p\frac{dN}{dt\,dx\,dp} \quad . \tag{15}$$

(The factors of 2 in (14) and (15) account for the contributions of the antiparticles.) Inserting Eq. (12) into Eq. (13) reduces the system to a single equation

$$\frac{d^2\tilde{A}}{d\tau^2} = \frac{e^2}{\pi m^2}\int_0^\tau d\tau'\frac{\tilde{A}(\tau')-\tilde{A}(\tau)}{\sqrt{\left[\tilde{A}(\tau')-\tilde{A}(\tau)\right]^2+1}}\left|\tilde{E}(\tau')\right|\log\left[1+\exp\left(-\frac{\pi}{\left|\tilde{E}(\tau')\right|}\right)\right] \tag{16}$$

$$+\frac{e^2}{\pi m^2}\,\mathrm{sign}\left(\tilde{E}(\tau)\right)\log\left[1+\exp\left(-\frac{\pi}{\left|\tilde{E}(\tau)\right|}\right)\right] \quad .$$

in terms of the dimensionless variables $\tilde{A} \equiv eA/m$, $\tilde{E}$, and τ.

The time evolution of $\tilde{E}$ and $\tilde{\jmath}$ is shown in the dashed curves of Fig. 1, where we see that for an initial field $\tilde{E}_{t=0} = 1$ there is good quantitative agreement between the results obtained with the two very different methods. The oscillations are faster and the electric fields decay more rapidly in the semiclassical calculation than in the Boltzmann-Vlasov model. This is because in addition to spontaneous pair creation, the quantum theory takes into account pair creation via bremsstrahlung ("induced" pair creation) and Bose-Einstein effects, all of which are neglected in the kinetic theory. Fig. 2 reflects this feature by comparing the plateau currents (the peak values in the first oscillation) as functions of the initial electric field. Decay of the oscillations would presumably be accelerated by the inclusion of collisions, which can be done by including a ϕ^4 term in the semiclassical equation (2) and the corresponding collision integral in the Boltzmann-Vlasov equation (11).

The distribution function $f(p,t)$, measured after several plasma oscillations, may be compared to the quantum theory's $V^{-1}dN/dk$ after the latter is smoothed, as shown in Fig. 3. Naturally, the curves have different normalizations and a relative displacement due to the slightly different values of j and A. Because of the similarity between the results in Fig. 3(b), one can use the kinetic theory model to explain various features of the particle distribution, such as the sharp edges and tails.

The kinetic theory can be improved by use of a source term that takes stimulated pair creation into account. Thus, we replace the right hand side of (11) with

$$\frac{dN}{dt\,dx\,dp} = (2f(p,t)+1)\frac{|eE(t)|}{2\pi}\log\left[1+\exp\left(-\frac{\pi m^2}{|eE(t)|}\right)\right]\,\delta(p) \quad . \tag{17}$$

With this source term the agreement between the kinetic theory and the quantum theory is even more striking, as demonstrated by the dashed-dotted line in Fig. 1(b).

A more extended comparison among the semiclassical and Boltzmann-Vlasov models will be presented elsewhere. We conclude that the phenomenological model with the stimulated emission source term yields a surprisingly good approximation to the semiclassical QED results.

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FIGURE CAPTIONS

FIGURE 1. (a) Time evolution of scaled electric field $\tilde{E}$ and current $\tilde{j}$, with initial value $\tilde{E} = 1.0$ and coupling $e^2/m^2 = 0.1$. Solid line is semiclassical scalar QED, and dashed line is Boltzmann-Vlasov model. (b) Same as (a), but for $\tilde{E} = 4.0$. The dashed-dotted curve is the Boltzmann-Vlasov model with the stimulated pair creation correction, Eq. (17).

FIGURE 2. Scaled current $\tilde{j}$ at the peak of the first oscillation, as a function of the initial scaled electric field $\tilde{E}$. Crosses are for semiclassical scalar QED, and circles are for Boltzmann-Vlasov model.

FIGURE 3. (a) Momentum distribution of produced pairs, for the evolution shown in Fig. 1(a), at time $\tau = 550$. The abscissa is the scaled kinetic momentum $\tilde{p} \equiv \tilde{k} - \tilde{A}$, with $\tilde{k} \equiv k/m$. (b) Data of (a) after smoothing (solid curve), compared with Boltzmann-Vlasov model (dashed curve).