

# Spontaneous symmetry breaking in strong-coupling lattice QCD at high density

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## Abstract

We determine the patterns of spontaneous symmetry breaking in strong-coupling lattice QCD in a fixed background baryon density. We employ a next-nearest-neighbor fermion formulation that possesses the  $SU(N_f) \times SU(N_f)$  chiral symmetry of the continuum theory. We find that the global symmetry of the ground state varies with  $N_f$  and with the background baryon density. In all cases the condensate breaks the discrete rotational symmetry of the lattice as well as part of the chiral symmetry group.

## I. INTRODUCTION

The study of quantum chromodynamics at high baryon density, nearly as old [1] as the theory itself, has gained impetus in recent years with new interest in the idea of color superconductivity (CSC) [2–5]. Recent work has led to a rich phase structure, including the possibilities of color-flavor locking [6] and crystalline superconductivity [7]. For a review see [8].

All this work depends on effective theories derived from (or at least motivated by) weak-coupling QCD. The running coupling, however, will become weak only at high densities; in fact it turns out that reliable calculations demand extremely high densities [9, 10]. If any of the predictions for moderate densities are to be believed, they must be confirmed by non-perturbative methods, or at least by models that incorporate QCD’s strong-coupling features.

In a previous paper [11] we constructed a framework for introducing baryons to lattice QCD at strong coupling. Though the lattice theory is a way of defining QCD at the most fundamental level, this process requires weak coupling; the strong-coupling theory may be regarded as an effective theory at large distances. It displays the non-perturbative effects of color confinement and spontaneous breakdown of chiral symmetry (for suitably chosen fermion formulations). Our framework is based on the Hamiltonian approach. We use strong coupling perturbation theory to write an effective Hamiltonian for color singlet objects [12, 13]. At lowest order we get an antiferromagnetic Hamiltonian that describes meson physics with a fixed baryon background distribution. (Baryons move only at higher order.) We study this Hamiltonian through its path integral, which takes the form of a nonlinear  $\sigma$  model.

The strong-coupling theory has no color degrees of freedom. Its properties can be determined through study of its global symmetries. The global symmetry group of the action depends on the formulation of the lattice fermions. For naive, nearest-neighbor (NN) fermions the symmetry is  $U(N)$  with

$$N = 4N_f, \tag{1}$$

realized on the quark spinors by combining the Dirac index with the flavor index. This too-large symmetry is indicative of the doubling problem of naive fermions [14]. Adding longer-range terms to the fermion kernel can reduce this artificial symmetry. We add a next-nearest-neighbor (NNN) term inspired by the SLAC fermion formulation [12, 15]. In weak coupling, of course, this theory is still doubled; in strong coupling, there are no apparent ill effects of this doubling, and the  $U(N)$  symmetry is broken to the  $U(N_f) \times U(N_f)$  symmetry of continuum QCD.<sup>1</sup>

## II. EFFECTIVE HAMILTONIAN AND $\sigma$ MODEL

In [11] we studied the NN theory only, and showed that its global  $U(N)$  symmetry is spontaneously broken to a subgroup that depends on the baryon number. In this paper we

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<sup>1</sup> The axial  $U(1)$  symmetry of the lattice theory is of course not present in continuum QCD, but it is inevitable on the lattice if we start with a local, chirally symmetric fermion theory [14]. We can mend our effective theory by hand, by adding new terms derived from an ’t Hooft instanton vertex.

add the NNN terms to the Hamiltonian and thus study a theory with the same symmetry as the continuum theory. The effective Hamiltonian in strong coupling is [11]

$$H = J_1 \sum_{\mathbf{n}, \eta} Q_{\mathbf{n}}^{\eta} Q_{\mathbf{n}+\hat{i}}^{\eta} + J_2 \sum_{\mathbf{n}, \eta} Q_{\mathbf{n}}^{\eta} Q_{\mathbf{n}+2\hat{i}}^{\eta} s_i^{\eta}. \quad (2)$$

Here  $Q_{\mathbf{n}}^{\eta}$  are  $U(N)$  charges at site  $\mathbf{n}$ , with  $\eta = 1, \dots, N^2$ . This Hamiltonian moves mesonic excitations around the lattice, leaving the baryon density fixed. The states at  $\mathbf{n}$  comprise a  $U(N)$  representation whose Young tableau has  $N_c$  columns. The number of rows  $m$  depends on the baryon number  $B$  at  $\mathbf{n}$  according to

$$m = B + 2N_f. \quad (3)$$

The sign factors  $s_i^{\eta}$  are given by

$$s_i^{\eta} = 2 \text{Tr} M^{\eta} \alpha_i M^{\eta} \alpha_i. \quad (4)$$

Here  $M^{\eta}$  are the matrices of the fundamental representation of the  $U(N)$  algebra, normalized in the usual way,  $\text{Tr} M^{\eta} M^{\eta'} = \frac{1}{2} \delta_{\eta\eta'}$ . The matrices  $\alpha_i$  are the  $4 \times 4$  Dirac matrices times the unit matrix in flavor space.

The NN term in the Hamiltonian is that of a  $U(N)$  antiferromagnet, while the NNN terms break the symmetry to  $U(N_f)_L \times U(N_f)_R$ . If one derives the fermion Hamiltonian by truncating the SLAC Hamiltonian, then both couplings  $J_1$  and  $J_2$  are positive, and  $J_2 = J_1/8$ . If we argue, however, that the strong-coupling Hamiltonian is derived by block-spin transformations applied to a short-distance Hamiltonian, then we cannot say much about the couplings that appear in it. We will assume that couplings in the effective Hamiltonian fall off strongly with distance, that is,  $0 < J_2 \ll J_1$ .

We use spin-coherent states [16] to write the partition function for the effective Hamiltonian (2). This is the path integral for a Euclidean nonlinear  $\sigma$  model. The  $\sigma$  field at site  $\mathbf{n}$  is an  $N \times N$  hermitian, unitary matrix that represents an element of the coset space  $U(N)/[U(m) \times U(N-m)]$ . It can be written as a unitary rotation of the reference matrix  $\Lambda$ ,

$$\sigma_{\mathbf{n}} = U_{\mathbf{n}} \Lambda U_{\mathbf{n}}^{\dagger}, \quad (5)$$

where

$$\Lambda = \begin{pmatrix} \mathbf{1}_m & 0 \\ 0 & -\mathbf{1}_{N-m} \end{pmatrix}. \quad (6)$$

and  $U_{\mathbf{n}} \in U(N)$ .

The action of the  $\sigma$  model is

$$S = \frac{N_c}{2} \int d\tau \left[ - \sum_{\mathbf{n}} \text{Tr} \Lambda U_{\mathbf{n}}^{\dagger} \partial_{\tau} U_{\mathbf{n}} + \frac{J_1}{2} \sum_{\mathbf{n}, i} \text{Tr} (\sigma_{\mathbf{n}} \sigma_{\mathbf{n}+\hat{i}}) + \frac{J_2}{2} \sum_{\mathbf{n}, i} \text{Tr} (\sigma_{\mathbf{n}} \alpha_i \sigma_{\mathbf{n}+2\hat{i}} \alpha_i) \right]. \quad (7)$$

The NN term is invariant under the global  $U(N)$  transformation  $U \rightarrow VU$ , or  $\sigma \rightarrow V\sigma V^{\dagger}$ . The NNN term is only invariant if  $V^{\dagger} \alpha_i V = \alpha_i$  for all  $i$ . This restricts  $V$  to the form

$$V = \exp [i (\theta_V^a + \gamma_5 \theta_A^a) \lambda^a], \quad (8)$$

where  $\lambda^a$  are flavor generators. This is a chiral transformation in  $U(N_f) \times U(N_f)$ .

The NNN term couples (discrete) rotational symmetry to the internal symmetry, *viz.*

$$\sigma_{\mathbf{n}} \rightarrow R^\dagger \sigma_{\mathbf{n}'} R, \quad \mathbf{n}' = \mathcal{R}\mathbf{n}. \quad (9)$$

Here  $\mathcal{R}$  is a  $90^\circ$  lattice rotation and  $R$  represents it according to

$$R = \exp \left[ i \frac{\pi}{4} \begin{pmatrix} \sigma_j & 0 \\ 0 & \sigma_j \end{pmatrix} \right] \otimes \mathbf{1}_{N_f}. \quad (10)$$

### III. NEAREST-NEIGHBOR THEORY

The overall  $N_c$  factor in Eq. (7) allows a systematic treatment in orders of  $1/N_c$ . In leading order, the ground state is found by minimizing the action, which gives field configurations that are  $\tau$  independent and that minimize the interaction terms. For the NN theory, minimizing the single link interaction

$$E = \frac{J_1}{2} \text{Tr} \sigma_1 \sigma_2 \quad (11)$$

allows us to construct the vacuum by placing  $\sigma_1$  and  $\sigma_2$  on the even and odd sites.

We impose a uniform baryon density,  $B_{\mathbf{n}} = B > 0$ , on the effective Hamiltonian by setting a fixed value of  $m > 2N_f$  on every site. To minimize the single-link energy (11) we first choose a basis where

$$\sigma_1 = \Lambda = \begin{pmatrix} \mathbf{1}_m & 0 \\ 0 & -\mathbf{1}_{N-m} \end{pmatrix}. \quad (12)$$

The analysis in [11] then shows that  $\sigma_2$  can take any value of the form

$$\sigma_2 = \begin{pmatrix} \sigma^{(m)} & 0 \\ 0 & \mathbf{1}_{N-m} \end{pmatrix}, \quad (13)$$

where the  $m \times m$  submatrix  $\sigma^{(m)}$  can be chosen freely in the submanifold  $U(m)/[U(2m-N) \times U(N-m)]$  according to

$$\sigma^{(m)} = U^{(m)} \Lambda^{(m)} U^{(m)\dagger}, \quad (14)$$

with

$$\Lambda^{(m)} = \begin{pmatrix} \mathbf{1}_{2m-N} & 0 \\ 0 & -\mathbf{1}_{N-m} \end{pmatrix} \quad (15)$$

and  $U^{(m)} \in U(m)$ . As mentioned, we construct a ground state of the infinite lattice by replicating  $\sigma_1$  and  $\sigma_2$  on the even and odd sites of the lattice. Thus while all the  $\sigma_{\mathbf{n}}$  on the even sites point to  $\Lambda$ , on the odd sites each  $\sigma_{\mathbf{n}}$  wanders independently in the submanifold. This classical ground state has a huge degeneracy, exponential in the volume.

In Ref. 11 we showed that the  $O(1/N_c)$  fluctuations generate a ferromagnetic interaction among the odd sites, causing them to align to a common value (“order from disorder” [17]). This is a Néel structure, with two sublattices. The even sites break  $U(N)$  to  $U(m) \times U(N-m)$  and then the odd sites break the symmetry further to  $U(2m-N) \times U(N-m) \times U(N-m)$ .

#### IV. NEXT-NEAREST-NEIGHBOR THEORY

Now we add the NNN interactions to the effective action. At the classical level, they do not by themselves remove the classical degeneracy of the NN theory. We have to introduce the  $O(1/N_c)$  fluctuations *first* in order to stabilize the Néel ground state of the NN Hamiltonian. Hence we assume  $1/N_c > J_2/J_1$ , and treat the NNN interactions as a perturbation that lifts part of the (global) degeneracy of the  $O(1/N_c)$  ground state.

We begin, then, by assuming a Néel ansatz that minimizes the NN term in the action (7). The NNN term acts within each of the two sublattices. Writing  $\sigma_{e,o}$  for the sublattice fields, the NNN contribution to the energy per  $2 \times 2 \times 2$  lattice cell is

$$E_{nnn} = \frac{J_2}{2} \sum_{a=e,o} \sum_i \text{Tr} [\sigma_a \alpha_i \sigma_a \alpha_i]. \quad (16)$$

Here  $\sigma_a$  is a global unitary rotation of the solution to the NN theory given by Eqs. (12)–(15).

We can find a lower bound for  $E_{nnn}$ . Writing in each term  $\Sigma_1^a = \sigma_a$  and  $\Sigma_2^{ai} = \alpha_{ai} \sigma_a \alpha_{ai}^\dagger$ , we have

$$E_{nnn} = \frac{J_2}{2} \sum_{ai} \text{Tr} \Sigma_1^a \Sigma_2^{ai}. \quad (17)$$

Note that  $\Sigma_{1,2}$  are unitary rotations of the reference matrix  $\Lambda$ . Each term in Eq. (17) may be bounded from below by allowing these unitary rotations to vary independently over the entire  $U(N)$  group. This is just the minimization problem posed in Eq. (11) above. The solution is given by Eqs. (12)–(15), whence the bound

$$E_{nnn} > 3J_2(4m - 3N). \quad (18)$$

We minimize  $E_{nnn}$  by writing an ansatz for  $\sigma_a$  that saturates the lower bound. At this point we choose to work in a basis where  $\gamma_5$  is diagonal,

$$\gamma_5 = \begin{pmatrix} \mathbf{1}_{N/2} & 0 \\ 0 & -\mathbf{1}_{N/2} \end{pmatrix}. \quad (19)$$

Our ansatz is

$$\begin{aligned} \sigma_e &= U \Lambda_e U^\dagger = U \begin{pmatrix} \mathbf{1}_m & 0 \\ 0 & -\mathbf{1}_{N-m} \end{pmatrix} U^\dagger, \\ \sigma_o &= U \Lambda_o U^\dagger = U \begin{pmatrix} \mathbf{1}_{2m-N} & 0 & 0 \\ 0 & -\mathbf{1}_{N-m} & 0 \\ 0 & 0 & \mathbf{1}_{N-m} \end{pmatrix} U^\dagger. \end{aligned} \quad (20)$$

(Note that  $\Lambda_o$  is a rotation of  $\Lambda_e$ .) This is a global rotation (via  $U$ ) of a particular configuration that minimizes the NN energy, as seen above. We further suppose that  $U$  takes the form

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} u & u \\ -u & u \end{pmatrix}, \quad (21)$$

with  $u \in U(N/2)$ .

The minimization of the energy via the ansatz is a problem of vacuum alignment [18]. We begin with the unperturbed NN problem, where the symmetry group  $G \equiv U(N)$  is broken to

$H \equiv U(2m - N) \times U(N - m) \times U(N - m)$ . Our reference vacuum is given by  $\sigma_{e,o} = \Lambda_{e,o}$ , giving a specific alignment of  $H$  as the invariance group of this vacuum. We determine the rotation matrix  $U$  [within the ansatz (21)] that minimizes the energy of the perturbation  $E_{nnn}$ , which we write as

$$E_{nnn} = \frac{J_2}{2} \sum_{ai} \text{Tr} \Lambda_a \bar{\alpha}_i \Lambda_a \bar{\alpha}_i^\dagger, \quad (22)$$

where  $\bar{\alpha}_i = U^\dagger \alpha_i U = \bar{\alpha}_i^\dagger$ .

We denote the generators of  $H$  collectively as  $T$  and the remaining generators of  $G$  as  $X$ . The  $T$  matrices commute with both  $\Lambda_e$  and  $\Lambda_o$ , while the  $X$  matrices do not. The symmetry-breaking term in the energy is  $E_{nnn}$ , given by Eq. (22) in terms of the rotated Hermitian matrices  $\bar{\alpha}_i$ . We project each  $\bar{\alpha}_i$  onto the  $T$  and  $X$  subspaces, giving the decomposition

$$\bar{\alpha}_i = \bar{\alpha}_i^T + \bar{\alpha}_i^X. \quad (23)$$

Using the invariance of  $\Lambda_{e,o}$  under  $T$ ,

$$[\Lambda_a, \bar{\alpha}_i^T] = 0, \quad (24)$$

and the orthogonality of the  $T$  and  $X$  subspaces,

$$\text{Tr} [\bar{\alpha}_i^T \bar{\alpha}_i^X] = 0, \quad (25)$$

we have

$$E_{nnn} = \frac{J_2}{2} \sum_{i,a} \text{Tr} \left[ (\bar{\alpha}_i^T)^2 + \Lambda_a \bar{\alpha}_i^X \Lambda_a \bar{\alpha}_i^X \right] \quad (26)$$

Following the block form of  $\Lambda_{e,o}$ , we divide the broken generators  $X$  into 3 sets, denoting them as  $X_a$  with  $a = 1, 2, 3$ . Their structures are respectively

$$\begin{pmatrix} 0 & \tilde{X}_1 & 0 \\ \tilde{X}_1^\dagger & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & \tilde{X}_2 \\ 0 & 0 & 0 \\ \tilde{X}_2^\dagger & 0 & 0 \end{pmatrix}, \quad \text{and} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \tilde{X}_3 \\ 0 & \tilde{X}_3^\dagger & 0 \end{pmatrix}. \quad (27)$$

Writing  $\bar{\alpha}_i^X = \sum_a \bar{\alpha}_i^{X_a}$ , the following relations can be proved easily:

$$\begin{aligned} [\Lambda_1, \bar{\alpha}_i^{X_1}] &= 0 & \{\Lambda_2, \bar{\alpha}_i^{X_1}\} &= 0 \\ \{\Lambda_1, \bar{\alpha}_i^{X_2}\} &= 0 & [\Lambda_2, \bar{\alpha}_i^{X_2}] &= 0 \\ \{\Lambda_1, \bar{\alpha}_i^{X_3}\} &= 0 & \{\Lambda_2, \bar{\alpha}_i^{X_3}\} &= 0. \end{aligned} \quad (28)$$

Using these together with the further orthogonality conditions,

$$\text{Tr} [\bar{\alpha}_i^{X_a} \bar{\alpha}_i^{X_b}] = 0, \quad a \neq b, \quad (29)$$

we bring Eq. (26) to the form

$$E_{nnn} = J_2 \sum_i \text{Tr} \left[ (\bar{\alpha}_i^T)^2 - (\bar{\alpha}_i^{X_3})^2 \right]. \quad (30)$$

The rotation  $U$  given in Eq. (21) saturates the lower bound for the energy. We will proceed to prove this for the case  $m \geq 3N/4$ . In view of Eq. (21) we can write  $\bar{\alpha}_i$  in the form

$$\bar{\alpha}_i = \begin{pmatrix} 0 & \bar{\sigma}_i \\ \bar{\sigma}_i & 0 \end{pmatrix}, \quad (31)$$

where  $\bar{\sigma}_i \equiv u^\dagger \sigma_i u$ . It is straightforward to check that for  $m \geq 3N/4$  we have

$$\bar{\alpha}_i^{X_3} = 0, \quad (32)$$

$$\bar{\alpha}_i^T = \begin{pmatrix} 0 & \bar{\sigma}'_i \\ \bar{\sigma}'_i^\dagger & 0 \end{pmatrix}, \quad (33)$$

with  $\bar{\sigma}'_i$  composed of the first  $(2m - 3N/2)$  columns of  $\bar{\sigma}_i$ ,

$$(\bar{\sigma}'_i)_{pq} = \begin{cases} (\bar{\sigma}_i)_{pq} & \text{for } q = 1, \dots, 2m - 3N/2, \\ 0 & \text{else.} \end{cases} \quad (34)$$

The energy is

$$E_{nnn} = 2J_2 \sum_{i,pq} |(\bar{\sigma}'_i)_{pq}|^2 = 2J_2 \sum_i \sum_{q=1}^{2m-3N/2} (\bar{\sigma}'_i^\dagger \bar{\sigma}'_i)_{qq} = 3J_2(4m - 3N), \quad (35)$$

which is exactly the lower bound.

According to Eq. (35), the bound is saturated for any  $u \in U(N/2)$ . Different choices of  $u$  are *not* in general related by transformations of the  $U(N_f) \times U(N_f)$  symmetry group. There is thus an accidental degeneracy of the vacuum when the NNN term is treated classically. This degeneracy is presumably lifted by fluctuations.

The simplicity of this calculation depends on the assumption  $m \geq 3N/4$ . For  $m < 3N/4$ , both  $\bar{\alpha}_i^{X_3}$  and  $\bar{\alpha}_i^T$  are nonzero. Moreover the index structure of the projections is more complex.<sup>2</sup> Therefore in these cases we resort to numerical minimization of Eq. (22) over  $u \in U(N/2)$ . In each case we find that the ansatz (20)–(21) yields a minimum that saturates the lower bound (18). Again, there is the possibility of accidental degeneracy.

Upon calculating the  $\sigma$  fields using Eqs. (20)–(21), it is straightforward to ascertain the symmetry of the vacuum. The  $U(N_f) \times U(N_f)$  generators that commute with both  $\sigma_e$  and  $\sigma_o$  form the unbroken subgroup of the NNN theory. The rest are broken generators that correspond to Goldstone bosons. We summarize our results in Table I, and we note the following:

- In cases of accidental degeneracy, we show the largest unbroken symmetry attainable. For  $m \geq 3N/4$  (i.e.,  $B \geq N_f$ ), this comes of the choice  $u = \mathbf{1}_{N/2}$ . For  $m < 3N/4$  our numerical work cannot rule out vacua with yet larger symmetry.
- Since the baryon background is fixed, we cannot tell whether the  $U(1)$  corresponding to baryon number is broken. [The  $U(1)_B$  group acts trivially on  $Q_n^\eta$  and on  $\sigma_n$ .]
- For each value of  $N_f$ , the case  $B = 2N_f$  is a completely saturated lattice. Each site is in a singlet under the chiral group, and there is no spontaneous symmetry breaking.

<sup>2</sup> An exception is the  $m = N/2$  case ( $B = 0$ ), which was solved in [11].

TABLE I: Breaking of  $SU(N_f)_L \times SU(N_f)_R \times U(1)_A$  for all baryon densities (per site) accessible for  $N_f \leq 3$ . Results for  $B = 0$  are from [11].

$N_f$	$ B $	Unbroken symmetry	Broken charges
1	0	–	1
	1	–	1
	2	$U(1)_A$	0
2	0	$SU(2)_V$	4
	1	$U(1)_{I_3}$	6
	2	$SU(2)_V$	4
	3	$U(1)_{I_3}$	6
	4	$SU(2)_L \times SU(2)_R \times U(1)_A$	0
3	0	$SU(3)_V$	9
	1	$U(1)_Y \times SU(2)_V$	13
	2	$U(1)_Y$	16
	3	$SU(3)_V$	9
	4	$U(1)_{I_3} \times U(1)_Y$	15
	5	$U(1)_{I_3} \times U(1)_Y \times U(1)_{A'}$	14
	6	$SU(3)_L \times SU(3)_R \times U(1)_A$	0

- The axial  $U(1)$  is not a symmetry of the continuum, and must be broken by hand. Wherever it appears in Table I it should be neglected, whether as an unbroken symmetry or as a broken charge.
- The  $U(1)_{A'}$  appearing in the table for  $N_f = 3$  is not the original  $U(1)_A$  group but rather is generated by  $\gamma_5 \otimes \lambda'$  with

$$\lambda' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (36)$$

This is the only case where an axial symmetry survives spontaneous symmetry breaking. If  $U(1)_A$  is broken by hand, so is  $U(1)_{A'}$ .

For all nonzero densities short of saturation the vacuum breaks rotational invariance. This is easily checked for  $m \geq 3N/4$  by noting that the ansatz for  $\sigma_{e,o}$  fails to commute with the rotation operator (10). For the other cases this is easily checked numerically. (In some cases a discrete symmetry around the  $z$  axis remains unbroken.) Since this is not a continuous symmetry it will not give rise to additional Goldstone bosons. The broken rotational invariance will of course affect the excitation spectrum. In particular, whereas the NN theory possesses excitations with linear and quadratic dispersion relations [17], the NNN theory will produce interesting admixtures with anisotropic dispersion relations, like those seen in [19]. We defer discussion of the excitations to a future publication.

## V. DISCUSSION

In comparing our results to those of continuum CSC calculations, one must keep in mind that we study systems with large, fixed, and discrete values of  $B$ , rather than with large, continuous  $\mu$ . Moreover, we use large- $N_c$  approximations which necessarily ignore the discrete properties of the  $SU(N_c)$  group that are essential to baryons. Quantum effects at finite  $N_c$ , treated correctly, should yield effects that are not accessible through the  $1/N_c$  expansion.

The values of  $(N_c, N_f)$  that are of interest for CSC are  $N_c = 3$  and  $N_f = 2$  or  $3$ . In the two-flavor case the favored  $qq$  condensate is a flavor singlet and a color triplet, so that while color is partially broken, chiral symmetry is unbroken. We do not see this for any density at  $N_f = 2$ . Plainly our results are due to a  $\bar{q}q$  condensate; whether there is a  $qq$  condensate as well cannot be ascertained.

For  $N_f > 2$  the situation is similar. Schäfer [20] has considered the color-flavor structure of the condensates that arise for  $N_c = 3$  and  $N_f \geq 3$ , and he has found that both color and flavor are partially broken, with a condensate that locks one or more subgroups of the flavor group to the color group. Since we work at large  $N_c$ , we should stand the argument on its head. A plausible  $qq$  condensate would lock successive subgroups of the *color* group to the flavor group and hence to each other, leaving unbroken the diagonal  $SU(N_f)_{L+R+C}$  and some leftover color symmetry. Judging by the global symmetry of the vacuum, perhaps we see this for  $(N_f = 3, B = 3)$ . The other cases could conceivably arise from a combination of  $\bar{q}q$  and  $qq$  condensates, but whether the latter actually occur is an open question.

Finally we note that according to diagrammatic power-counting arguments [21, 22], CSC should disappear in the 't Hooft limit ( $N_c \rightarrow \infty$ ,  $g^2 N_c$  fixed). We do not strictly work in this limit, since the large- $N_c$  approximation is applied only to the effective strong-coupling Hamiltonian, after  $g^2$  has disappeared into setting the energy scale.

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