Chiral Transition of SU(4) Gauge Theory with Fermions in Multiple Representations

Venkitesh Ayyar[1], *Thomas* DeGrand[1], *Daniel C.* Hackett[1]⋆, *William I.* Jay[1], *Ethan T.* Neil[1,2,], *Yigal* Shamir[3], and *Benjamin* Svetitsky[3]

[1] *Department of Physics, University of Colorado, Boulder, Colorado 80309, USA*
[2] *RIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, USA*
[3] *Raymond and Beverly Sackler School of Physics and Astronomy, Tel Aviv University, 69978 Tel Aviv, Israel*

Abstract. We report preliminary results on the finite temperature behavior of SU(4) gauge theory with dynamical quarks in both the fundamental and two-index antisymmetric representations. This system is a candidate to present scale separation behavior, where fermions in different representations condense at different temperature or coupling scales. Our simulations, however, reveal a single finite-temperature phase transition at which both representations deconfine and exhibit chiral restoration. It appears to be strongly first order. We compare our results to previous single-representation simulations. We also describe a Pisarski-Wilczek stability analysis, which suggests that the transition should be first order.

1 Introduction

Lattice gauge theories with fermions in multiple representations ("multirep" theories) provide an arena to test the old ideas of tumbling or (for vectorlike systems) scale separation [1]. The physical picture is that when a gauge coupling becomes sufficiently strong in the infrared, a scalar fermion bilinear will form, breaking chiral symmetry. In a system with multiple representations of fermions, a weaker gauge coupling is needed to drive condensation for higher representation fermions, since their color charges are greater. Thus, different representations of fermion may condense at different scales. Quenched simulations from the early 80's (performed on small lattices with large gauge couplings) [2–5] appeared to show such behavior. However, quenching neglects the back-reaction of the fermions on the gauge dynamics. It may also happen that the scales for chiral symmetry breaking and deconfinement are different. Some old simulations with dynamical fermions (see [6]) indicate this behavior. The issue with these systems is that they all appear to be near or beyond the conformal window (whose precise boundary is still controversial), so they may not be chirally broken at all. (See refs. [7, 8] for reviews.)

Here, we study an SU(4) gauge theory with two flavors of Dirac fermions charged under the fundamental irreducible representation (irrep) F (quartet, **4**) of SU(4); and an additional two flavors of Dirac fermions charged under the two-index antisymmetric irrep A_2 (sextet, **6**) of SU(4). The first- and second-order coefficients of the beta function are negative, signaling that this system is likely to

---

⋆Speaker, e-mail: daniel.hackett@colorado.edu

be an ordinary confining and chirally broken system at zero temperature. Our simulations show that is the case [9]. In these Proceedings, we describe our preliminary results for the finite-temperature phase structure of this theory.

2 Lattice Details

The gauge action used in our simulations is the "nHYP dislocation suppressing" (NDS) action [10]. This is a sum of the usual plaquette term plus an additional term made from smeared links. We scale all new parameters together, leaving β the only free parameter in the gauge sector. The fermions use the clover action. The links have nHYP smearing [11, 12]. The clover coefficient is taken to be $c_{sw} = 1$, a choice known to work well with nHYP smearing in QCD [13] and higher-irrep fermions [14]. We have a three-dimensional space of bare parameters: β and the hopping parameters κ_4 for the Fs and κ_6 for the A_2s. Spectroscopic quantities are measured as screening masses, that is, from correlators projected along one of the spatial directions of the lattice. When we quote a value for a quark mass, we mean the quantity extracted from the Axial Ward Identity, or AWI quark mass.

Simulations are performed on $12^3 \, times 6$ and $16^3 \times 8$ volumes. The two N_t's allow us to diagnose whether an observed crossover or transition is a finite temperature transition or a bulk one.

To diagnose separately whether each irrep of fermion is confined, we use the fundamental-irrep Polyakov loop and the A_2 higher-representation Polyakov loop. The presence of dynamical fundamental fermions mean neither Polyakov loop is an exact order parameter for the breaking of center symmetry. Physically, a Polyakov loop in some representation measures the free-energy cost of a static charge of that irrep. So, we still expect the Polyakov loop to jump from a small value to a large one when the corresponding irrep of charge deconfines. We thus associate a change in the Polyakov loops with the words "confinement-deconfinement transition."

To determine whether chiral symmetry is broken for an irrep, we should check for the presence of the corresponding chiral condensate. Because we are using Wilson fermions the condensates are difficult to access directly, so we instead use chiral parity doubling as an indirect probe of whether a condensate has formed for either irrep. In the chirally-restored phase, the GMOR relation ($m_{PS}^2 \propto m_q$ is lost and the pseudoscalar mass plateaus at $m_P = \pi T$, while the parity-partner scalar and pseudoscalar meson states, and the vector and pseudovector meson states, become nearly degenerate. This degeneracy disappears when chiral symmetry is broken.

More details on our phase diagnostics are published in a companion proceedings [15].

3 Results: Antisymmetric-Only Limit

To complement our investigation of the full (both fermions dynamical) theory, we have also examined the phase structure of the one-representation limiting cases of our theory (with either the Fs or the A_2s infinitely massive and decoupled). Our group previously studied the A_2-only theory using the same action without the NDS term [16]. These results allow us to diagnose the effects of the new term.

Figure 1 shows our new findings for the phase structure of the A_2-only theory. We find that the confinement and chiral transitions coincide everywhere that we have investigated, as seen in typical slices through bare parameter space as in figure 2. The upper panel shows gauge observables (Polyakov loops in the two representations and the spatial-temporal anisotropy in the Wilson flow [15, 17–19]). The middle panel shows the scalar-pseudoscalar and vector-axial vector mass differences. They both show a step at the same value of κ_6. The bottom panel shows the AWI quark mass, which is not much affected at the crossover, and the plaquette, which orders.
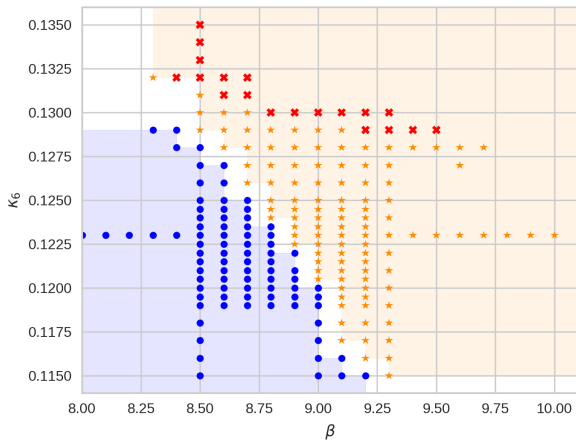
Figure 1. Phase diagram for the A_2-only theory with the NDS term, from the present study. All data from lattices with $N_t = 6$. Blue dots indicate confined ensembles where $m_{q6} > 0$. Yellow stars (red Xs) indicate deconfined ensembles with $m_{q6} > 0$ ($m_{q6} < 0$). Blue regions are unambiguously confined, orange regions are unambiguously deconfined, and white regions are phase-ambiguous (transition lives here).
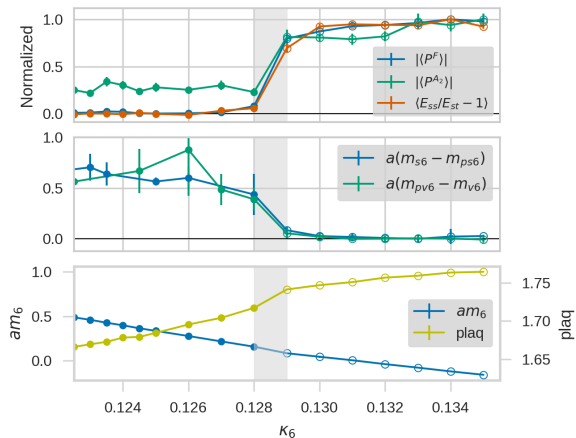

Figure 2. Polyakov loops (unflowed) and flow anisotropy (measured at $t/a^2 = 1$) [15, 17–19] (top), chiral doubling observables (middle), and quark mass and plaquette (bottom) varying κ_6 along a slice of constant $\beta = 8.5$ on $N_t = 6$ in the A_2-only theory with the NDS action.

Our previous study, with a Wilson gauge action, saw an additional first-order bulk transition; essentially all observables jumped as we crossed it. With the NDS action, we see no evidence for such a transition where we have looked.

A high order Pisarski-Wilczek-style calculation [20, 21] finds a stable fixed point for this theory and thus predicts that the chiral transition may be second order. We observe a smooth crossover, consistent with (but not demonstrative of) this prediction. This is also consistent with the previous study of this system [16].

4 Results: Multirep Phase Structure

We collected data for the multirep theory on $12^3 \times 6$ and $16^3 \times 8$ lattices at 21 values of the bare gauge coupling and many (typically $O(10)$) values of (κ_4, κ_6) per β value. We explored $\beta = 7.4$ and $\beta = 7.75$ more extensively, with 159 and 236 ensembles respectively.

The strong coupling phase of our system is confined and both representations of fermions are chirally condensed. Everywhere that we have investigated, we observe a single thermal phase transition at which both irreps deconfine and chiral symmetry for both species of fermion is restored. Figure 3 shows the behavior of all of our phase diagnostics on a typical slice through bare parameter space. It
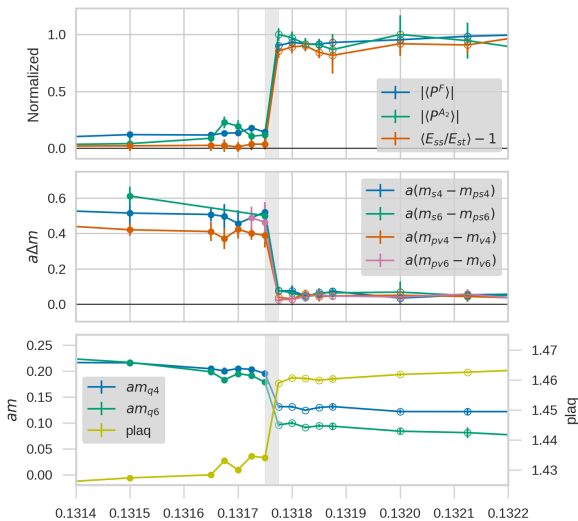
Figure 3. Slice varying κ_6 while holding $\beta = 7.4$ and $\kappa_4 = 0.1285$ constant on $N_t = 6$. The gray band brackets the transition. Closed (open) dots indicate that the ensemble is confined (deconfined) as shown by the long flow time Polyakov loops test [15]. **Top:** Polyakov loops for both irreps and the flow anisotropy observable [17–19] respond simultaneously. Quantities are plotted as a proportion of their maximum value along the slice for ease of comparison of their qualitative behavior. **Center:** Mass splittings between parity partner mesonic states: scalar vs. pseudoscalar, and vector vs. pseudovector. The chiral transition occurs simultaneously for both irreps. **Bottom:** Lattice units quark masses for each irrep and plaquette. All quantities jump discontinuously at the transition.

shows results versus κ_6 across the transition at fixed β and κ_4. Simultaneously, both Polyakov loops acquire large expectation values and chiral doubling sets in, indicating that the chiral transitions and confinement transitions for each irrep all coincide.

In order to diagnose whether the transition encountered is a bulk transition, we have investigated two different temporal extents, $N_t = 6$ and $N_t = 8$. Figure 4 shows the resulting two $\kappa_4 - \kappa_6$ phase diagrams for $\beta = 7.75$. The phase-ambiguous region from the $N_t = 6$ diagram is overlaid on to the $N_t = 8$ diagram, demonstrating that the transition moves substantially as N_t is changed. This strong response to the change in temperature indicates the transition is a physical thermal transition. Decreasing the bare temperature $1/N_t$ at fixed bare parameters (and thus fixed a) decreases the physical temperature $T = 1/aN_t$. At lower physical temperature, the deconfinement transition will occur at lighter quark masses and thus greater κs for a given β.

The observed transition appears to be strongly first order. As can be seen in figure 3, all observables that we have investigated jump discontinuously at the phase transition. Further, we have observed several tunneling events during equilibration, indicating strong metastability in HMC time. These events can occur when we start equilibrating an ensemble in one phase using a configuration from an ensemble in the other phase as a seed. After $O(10 - 100)$ times longer than typical equilibration times for these lattices (sometimes after 1000 HMC trajectories), the system will suddenly tunnel to the target phase and all observables acquire values typical for the new phase. This is characteristic behavior for a first-order transition. The quark masses where we have investigated the transition are shown in figure 5.

5 Continuum Theory: Multirep Pisarski-Wilczek

To see whether the first order behavior we observe is an expected result, we performed a Pisarski-Wilczek [24] stability analysis of the chiral transition in the multirep theory. The calculation is an analysis of the critical behavior of an effective three-dimensional field theory of the two chiral condensates of the theory. If any infrared-stable fixed points exist for this theory, the transition can be second order if it occurs in the basin of attraction of that fixed point (but may still be first order otherwise). If no infrared-stable fixed points exist, the transition must be first order. To proceed, we
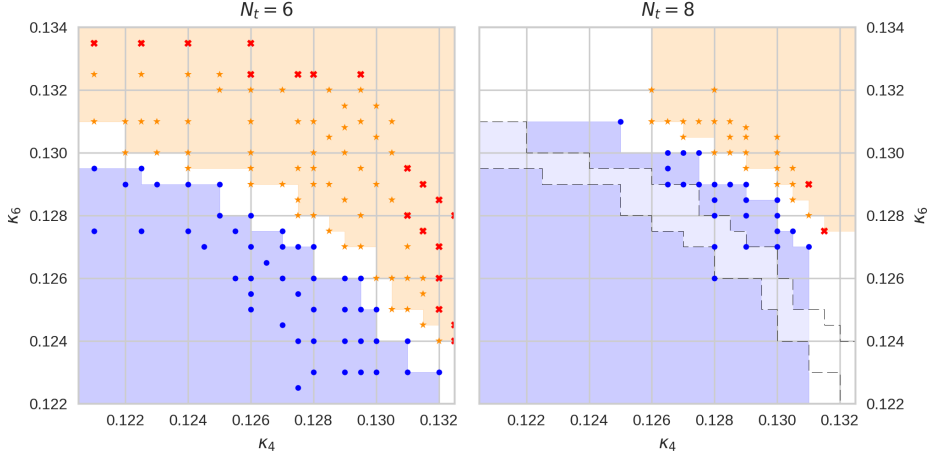
Figure 4. Phase diagram in the multirep theory at constant $\beta = 7.75$. At left is the diagram for $N_t = 6$ lattices, while at right is the same region of bare parameter space for $N_t = 8$ lattices. Blue dots indicate confined ensembles with $m_q > 0$ for both species. Yellow stars (red Xs) indicate deconfined ensembles with $m_q > 0$ for both ($m_q < 0$ for either) species. Blue regions are unambiguously confined, orange regions are unambiguously deconfined, and white regions are phase-ambiguous (transition lives here). The pale band overlaid on $N_t = 8$ is the phase-ambiguous region from $N_t = 6$, demonstrating that the transition moves as N_t is varied.
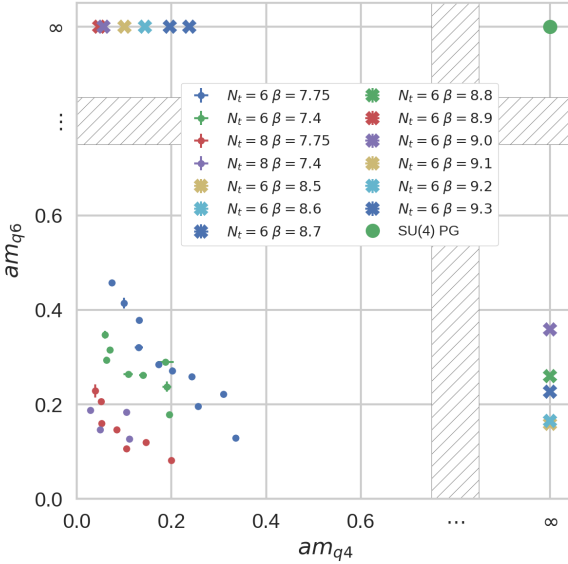

Figure 5. Multirep Columbia plot, by analogy with QCD [22]. Each point is an ensemble on the transition boundary, found by varying κs at some fixed β and N_t. Each color is associated with a different β and N_t; Circles indicate the transition encountered near this ensemble is first-order. Xs indicate the transition found is continuous. All quark masses are plotted in lattice units. Pure gauge SU(4) is known to be first order [23].

first identify the symmetries and spontaneous symmetry-breaking (SSB) pattern of our theory and use them to construct the most general Landau-Ginzburg-Wilson (LGW) Lagrangian, including only relevant and marginal terms. We compute the β functions for this effective theory to one loop using the ϵ expansion: we expand as usual in small $\epsilon = 4 - d$, but then set $\epsilon = 1$. Finally, we perform a stability analysis of the resulting β functions in the zero-mass theory (with relative signs of couplings constrained to induce the correct SSB pattern). The validity of the calculation is limited by both the ϵ expansion and by working to only one loop. Analyses which go to higher order in perturbation theory and treat three-dimensionality more cautiously can find stable fixed points that are missed by the approach used here [20, 21].

We begin by considering N_F Dirac flavors of F fermions and N_{A_2} Dirac flavors of A_2 fermions. Theories with fermions charged under n different irreps generically have n independent axial symmetries, which can be used to construct $n - 1$ non-anomalous axial symmetries. For our theory, there are two independent U(1)$_A$s. There is a non-anomalous linear combination of these two axial currents. Thus, there is a good U(1)$_A$ which is spontaneously broken, yielding a flavor-singlet axial pNGB. This U(1)$_A$ must be a good symmetry of our effective theory. [25, 26]

The F of SU(4) is complex and so the F sector has the typical chiral symmetry SU(N_F)$_L \times$ SU(N_F)$_R$. The A_2 of SU(4) is a real irrep, which expands the A_2-sector chiral symmetry group to SU($2N_{A_2}$) [27]. The SSB pattern of our theory is then

$$\text{SU}(N_F)_L \times \text{SU}(N_F)_L \times \text{SU}(2N_{A_2}) \times \text{U}(1)_A \to \text{SU}(N_F)_V \times \text{SO}(2N_{A_2}). \tag{1}$$

For the fundamental fermion fields q, define the complex $N_F \times N_F$ matrix field $\phi \sim \bar{q}_R^i q_{Lj}$ (where i and j are Dirac flavor indices). It transforms under chiral rotations like

$$\phi \to e^{2i\alpha_F} U_L \phi U_R, \tag{2}$$

where $U_L, U_R \in$ SU(N_F) and α_F is the angle of the axial rotation for the Fs [24]. For the two-index antisymmetric fermion fields Q, define the symmetric complex $2N_{A_2} \times 2N_{A_2}$ field $\theta \sim Q_I Q_J$ (where Q is a left-handed Weyl field, and I and J are Weyl flavor indices). It transforms under chiral rotations like

$$\theta \to e^{2i\alpha_{A_2}} V^T \theta V, \tag{3}$$

where $V \in$ SU($2N_{A_2}$) and α_{A_2} is the angle of the axial rotation [20, 21].

To construct the LGW Lagrangian, we write down all non-irrelevant terms that are invariant under equations (2) and (3). Including the single non-irrelevant term which couples F and A_2 fermions, the full Lagrangian is

$$\begin{aligned}
\mathcal{L}^{\text{Multirep}} = {} & \text{Tr}[\partial_\mu \phi^\dagger \partial^\mu \phi] + r_F \,\text{Tr}[\phi^\dagger \phi] + u_F (\text{Tr}[\phi^\dagger \phi])^2 + v_F \,\text{Tr}[(\phi^\dagger \phi)^2] \\
& + \text{Tr}[\partial_\mu \theta^\dagger \partial^\mu \theta] + r_{A_2} \,\text{Tr}[\theta^\dagger \theta] + u_{A_2} (\text{Tr}[\theta^\dagger \theta])^2 + v_{A_2} \,\text{Tr}[(\theta^\dagger \theta)^2] \\
& + w \,\text{Tr}[\phi^\dagger \phi] \,\text{Tr}[\theta^\dagger \theta].
\end{aligned} \tag{4}$$

In order to break the axial symmetry of the theory from U(1)$_A^{(F)} \times$ U(1)$_A^{(A_2)}$ to the non-anomalous U(1)$_A$, we must introduce all non-irrelevant terms constructed from determinants, which respect the unbroken U(1)$_A$. They are $\det \phi \to e^{2iN_F \alpha_F} \det \phi$ and $\det \theta \to e^{4iN_{A_2} \alpha_{A_2}} \det \theta$. For the exact flavor content of our theory ($N_F = N_{A_2} = 2$), we find that simultaneous axial rotations obeying $\alpha_F = -2\alpha_{A_2}$ are unbroken symmetries [26]. The lowest-order such term that respects this symmetry is $\delta\mathcal{L} \sim \det \phi \det \theta + (\text{c.c.})$, but because $[\det \phi] = N_F = 2$ and $[\det \theta] = 2N_{A_2} = 4$, this term is irrelevant for our theory. Thus, equation 4 is the final Lagrangian, with both U(1)$_A$s as good symmetries. Physically, we find that the axial anomaly does not play a role in the critical behavior of the theory.

We observe only one transition in our data, so we only consider the behavior of the theory when both irreps are simultaneously driven to criticality; thus, we set $r_F = r_{A_2} = 0$. We then compute the β functions of the resulting Lagrangian to one loop. With all of the couplings redefined by the same overall factor to absorb geometric constants, these are

$$\beta_{u_F} = -u_F + (N_F^2 + 4)u_F^2 + 4N_F u_F v_F + 3v_F^2 + 4N_{A_2}(2N_{A_2} + 1)w^2$$
$$\beta_{v_F} = -v_F + 6u_F v_F + 2N_F v_F^2$$
$$\beta_{u_{A_2}} = -u_{A_2} + \tfrac{1}{2}(4N_{A_2}^2 + 2N_{A_2} + 8)u_{A_2}^2 + 2(2N_{A_2} + 1)u_{A_2}v_{A_2} + \tfrac{3}{2}v_{A_2}^2 + 4N_F^2 w^2$$
$$\beta_{v_{A_2}} = -v_{A_2} + 6u_{A_2}v_{A_2} + (2N_{A_2} + \tfrac{5}{2})v_{A_2}^2$$
$$\beta_w = -w + w\left((N_F^2 + 1)u_F + 2N_F v_F + \tfrac{1}{2}(4N_{A_2}^2 + 2N_{A_2} + 4)u_{A_2} + (2N_{A_2} + 1)v_{A_2} + 2w\right).$$

A stability analysis amounts to finding the fixed points of these β functions and determining whether any of the eigenvalues of the stability matrix $\partial\beta_{g_i}/\partial g_j$ (where $g_i \in \{u_F, v_F, u_{A_2}, v_{A_2}, w\}$) are negative there. We find six fixed points, none of which are infrared-stable. Our calculation thus indicates that the transition in our theory should be first-order. This result is consistent with our data.

6 Conclusion

We do not observe any separation of phases: our lattice data indicate that both irreps confine and break chiral symmetry simultaneously. Our system does not show scale separation. Further, this transition appears to be strongly first-order, consistent with our multirep Pisarski-Wilczek calculation.

Although the finite-temperature properties of this model are not relevant for LHC phenomenology, they may have implications for cosmology. In particular, a first-order phase transition in the early universe would be expected to generate a primordial gravitational wave signature, see e.g. [28, 29].

References

[1] S. Raby, S. Dimopoulos, L. Susskind, Nucl. Phys. **B169**, 373 (1980)

[2] J.B. Kogut, M. Stone, H.W. Wyld, S.H. Shenker, J. Shigemitsu, D.K. Sinclair, Nucl. Phys. **B225**, 326 (1983)

[3] J.B. Kogut, M. Stone, H.W. Wyld, W.R. Gibbs, J. Shigemitsu, S.H. Shenker, D.K. Sinclair, Phys. Rev. Lett. **50**, 393 (1983)

[4] J.B. Kogut, J. Shigemitsu, D.K. Sinclair, Phys. Lett. **138B**, 283 (1984)

[5] J.B. Kogut, J. Shigemitsu, D.K. Sinclair, Phys. Lett. **145B**, 239 (1984)

[6] F. Karsch, M. Lutgemeier, Nucl. Phys. **B550**, 449 (1999), `hep-lat/9812023`

[7] T. DeGrand, Rev. Mod. Phys. **88**, 015001 (2016), `1510.05018`

[8] B. Svetitsky, *Looking behind the Standard Model with lattice gauge theory*, in *Proceedings, 35th International Symposium on Lattice Field Theory (Lattice2017): Granada, Spain*, to appear in EPJ Web Conf., `1708.04840`

[9] V. Ayyar, T. DeGrand, M. Golterman, D.C. Hackett, W.I. Jay, E.T. Neil, Y. Shamir, B. Svetitsky, work in progress

[10] T. DeGrand, Y. Shamir, B. Svetitsky, Phys. Rev. **D90**, 054501 (2014), `1407.4201`

[11] A. Hasenfratz, F. Knechtli, Phys. Rev. **D64**, 034504 (2001), `hep-lat/0103029`

[12] A. Hasenfratz, R. Hoffmann, S. Schaefer, JHEP **05**, 029 (2007), `hep-lat/0702028`

[13] C.W. Bernard, T.A. DeGrand, Nucl. Phys. Proc. Suppl. **83**, 845 (2000), `hep-lat/9909083`

[14] Y. Shamir, B. Svetitsky, E. Yurkovsky, Phys. Rev. **D83**, 097502 (2011), `1012.2819`

[15] V. Ayyar, D.C. Hackett, W.I. Jay, E.T. Neil, Y. Shamir, B. Svetitsky (TACO), *Confinement study of an SU(4) gauge theory with fermions in multiple representations*, in *Proceedings, 35th International Symposium on Lattice Field Theory (Lattice2017): Granada, Spain*, to appear in EPJ Web Conf.

[16] T. DeGrand, Y. Liu, E.T. Neil, Y. Shamir, B. Svetitsky, Phys. Rev. **D91**, 114502 (2015), `1501.05665`

[17] S. Datta, S. Gupta, A. Lytle, Phys. Rev. **D94**, 094502 (2016), `1512.04892`

[18] S. Datta, S. Gupta, A. Lytle, PoS **LATTICE2016**, 091 (2016), `1612.07985`

[19] M. Wandelt, F. Knechtli, M. Günther, JHEP **10**, 061 (2016), `1603.05532`

[20] F. Basile, A. Pelissetto, E. Vicari, JHEP **02**, 044 (2005), `hep-th/0412026`

[21] F. Basile, A. Pelissetto, E. Vicari, PoS **LAT2005**, 199 (2006), `hep-lat/0509018`

[22] P. Petreczky, J. Phys. **G39**, 093002 (2012), `1203.5320`

[23] S. Datta, S. Gupta, Phys. Rev. **D80**, 114504 (2009), `0909.5591`

[24] R.D. Pisarski, F. Wilczek, Phys. Rev. **D29**, 338 (1984)

[25] T.E. Clark, C.N. Leung, S.T. Love, J.L. Rosner, Phys. Lett. **B177**, 413 (1986)

[26] T. DeGrand, M. Golterman, E.T. Neil, Y. Shamir, Phys. Rev. **D94**, 025020 (2016), `1605.07738`

[27] M.E. Peskin, Nucl. Phys. **B175**, 197 (1980)

[28] P. Schwaller, Phys. Rev. Lett. **115**, 181101 (2015), `1504.07263`

[29] C. Caprini et al., JCAP **1604**, 001 (2016), `1512.06239`

[30] MILC Collaboration, http://www.physics.utah.edu/~detar/milc/