

Effective spin models for the confinement phase transition*

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Spatial correlations—bubbles, domain walls, etc.—can best be studied by concentrating on the degrees of freedom most relevant to the problem. For the finite temperature confinement transition, I integrate out all gauge degrees of freedom, leaving only spins—Ising or Potts—related to the Wilson line. I present problems that arise in the course of this transformation and some results for the effective spin action.

1. EFFECTIVE MODELS

The spatial structure of the $SU(3)$ gauge theory near the confinement phase transition presents many interesting problems. One might study the statics and dynamics of a planar boundary between the high- and low-temperature phases [1] as well as the nucleation and growth of bubbles and the possible stability of bubbles in either phase [2]. Straightforward lattice approaches to these properties are difficult because of the large lattices needed. We can make considerable progress by deriving an equivalent spin model to study [3]. Then, instead of working with a non-Abelian gauge theory on a periodic four-dimensional lattice of limited size, we can work on a three-dimensional spin model in a much larger spatial volume. The spin variables may also show domain structure at a glance that would be hard to discern in the gauge theory.

Defining new degrees of freedom, let us associate $\sigma = +1$ with one phase and $\sigma = -1$ with the other. Then a general effective spin model will have the form

$$S_{\text{spin}} = \sum f(\mathbf{n} - \mathbf{n}') \sigma_{\mathbf{n}} \sigma_{\mathbf{n}'} + (3 - \text{spin}) + (4 - \text{spin}) + \dots \quad (1)$$

Simple restrictions of this action are the next-

nearest-neighbor Ising model

$$S = \beta \sum_{\text{nn}} \sigma_{\mathbf{n}} \sigma_{\mathbf{n}'} + \gamma \sum_{\text{nnn}} \sigma_{\mathbf{n}} \sigma_{\mathbf{n}'} \quad (2)$$

(recall the anisotropic version, the ANNNI model [4]) and one of the simple models used to describe amphiphilic (oil–water–soap) mixtures [5],

$$S = J \sum_{\text{nn}} \sigma_{\mathbf{n}} \sigma_{\mathbf{n}'} + h \sum \sigma_{\mathbf{n}} + \gamma \sum_{\text{nnn}} \sigma_{\mathbf{n}} \sigma_{\mathbf{n}'} + \delta \sum_{\text{triples}} \sigma_{\mathbf{n}} \sigma_{\mathbf{n}'} \sigma_{\mathbf{n}''} \quad (3)$$

When the couplings in these well-studied actions are chosen to give competition between interactions, one finds a large variety of modulated equilibrium phases. It would be exciting to discover such physics in our gauge theory, but even if this doesn't happen, the advantages of the spin models over the gauge theory are obvious.

2. ISING ACTION

Let's define the Ising variable σ more precisely [3]. We begin with the Wilson line,

$$L_{\mathbf{n}} = \text{Tr} \prod_{\tau} U_{\mathbf{n},\tau}^0, \quad (4)$$

and define $\sigma_{\mathbf{n}} = \{-1, +1\}$ according to whether $|L_{\mathbf{n}}|$ is less than or greater than some parameter r . Refinements of this prescription might include smearing $L_{\mathbf{n}}$ first, or smearing and decimating (i.e., blocking).

Then we generate gauge configurations on an $N_t \times N_s^3$ lattice. Each gauge configuration gives a

*Talk given at Lattice '98, the International Symposium on Lattice Field Theory, Boulder, Colorado, 13–18 July 1998

†Supported by the Israel Science Foundation under Grant No. 255/96-1

spin configuration on an N_s^3 lattice. From these configurations, we calculate an effective action for σ . Choosing a set of operators \mathcal{O}^α , we write a (truncated) action

$$S_{\text{eff}}[\sigma] = \sum_{\alpha} \beta_{\alpha} \mathcal{O}^{\alpha} \quad (5)$$

and evaluate the coefficients β_{α} via the Schwinger-Dyson equations [6] of the spin model,

$$\langle \tilde{\mathcal{O}}_{\mathbf{n}}^{\alpha} \rangle = - \langle \tilde{\mathcal{O}}_{\mathbf{n}}^{\alpha} \exp 2\tilde{\mathcal{S}}_{\mathbf{n}} \rangle. \quad (6)$$

($\tilde{\mathcal{O}}_{\mathbf{n}}^{\alpha}$ is the piece of \mathcal{O}^{α} that contains the spin $\sigma_{\mathbf{n}}$, and similarly $\tilde{\mathcal{S}}_{\mathbf{n}}$.)

We expect that the effective couplings β_{α} will be continuous functions of the gauge coupling g . As g passes through g^* , the gauge theory undergoes its confinement phase transition. We expect that $\beta_{\alpha}^* = \beta_{\alpha}(g^*)$ will be values of the effective couplings at which the Ising model goes through a phase transition from $\langle \sigma \rangle < 0$ to $\langle \sigma \rangle > 0$.

The result of the calculation [3] frustrates these expectations. It turns out that the couplings β_{α} are *themselves* discontinuous at $g = g^*$ (which of course makes $\langle \sigma \rangle$ discontinuous as expected). Perhaps this isn't surprising: The couplings β_{α} are derived from measured correlation functions that are themselves discontinuous. In any case, the situation is reminiscent of a hoary controversy regarding the renormalization group transformation near a first-order transition. One school held that couplings in a blocked Hamiltonian must be continuous functions of the unblocked couplings, and the transition is created by a discontinuity fixed point. The other school held that unblocked couplings on either side of the transition will flow immediately to well-separated blocked couplings, creating the discontinuity immediately. The point was settled five years ago by theorems proved by van Enter, Fernández, and Sokal [7], which state in brief that discontinuities in the effective couplings (or the blocked couplings) are *impossible*.

To expand upon this a bit: For each configuration, we map $U_{\mathbf{n},\tau} \rightarrow L_{\mathbf{n}} \rightarrow \sigma_{\mathbf{n}}$. The original measure is $d\mu[U] = \exp -S_W[U] dU$, where S_W is *local*. We seek to replace this by a new measure $d\mu[\sigma]$. At a first order transition, observables are

discontinuous. Nevertheless, say the theorems, $d\mu[\sigma]$ *must* turn out continuous; otherwise $d\mu[\sigma]$ is non-Gibbsian, which means that if it is to be written as $\exp -S_{\text{eff}}[\sigma] d\sigma$, then S_{eff} will not exist in the infinite volume limit. The fault, of course, lies in the mapping. We need better spin variables.

3. POTTS ACTIONS

Perhaps the definition of the Ising spin σ may be modified to yield a valid effective action. The suspicion arises, however, that the simple projection used above neglects some essential physics characteristic of the phase transition, namely, the fact that it is a $Z(3)$ order-disorder transition [8].

One may focus on the $Z(3)$ physics by defining spins $P_{\mathbf{n}}$ that encode the phase of $L_{\mathbf{n}}$, projected to the $Z(3)$ directions. This was done by Fukugita, Okawa, and Ukawa [9] who considered a 3-state Potts model with a general two-spin action,

$$S_{\text{Potts}} = \sum f(|\mathbf{n} - \mathbf{n}'|) \delta(P_{\mathbf{n}}, P'_{\mathbf{n}}) \quad (7)$$

The couplings $f(|\mathbf{n} - \mathbf{n}'|)$ were found to decay satisfyingly with distance *and* to be continuous at the phase transition. This action deserves to be studied further, in particular to see whether it reproduces multi-spin correlations well.

The 3-state Potts variables do not, however, give a simple mapping of ordered and disordered regions as do the Ising variables. For this reason I have gone a step further, combining the Ising and Potts degrees of freedom to give a *four*-state model. I define $s_{\mathbf{n}} = 0$ if $|L_{\mathbf{n}}| < r$, and $s_{\mathbf{n}} = 1, 2, 3$ according to the phase of $L_{\mathbf{n}}$ when $|L_{\mathbf{n}}| > r$. The effective action is that of a 4-state model with the P_4 symmetry broken to P_3 , truncated to a magnetic field term plus eight two-spin couplings as shown in Table 1. I show the couplings β^* (preliminary, without error estimates) at the phase transition for $N_t = 2$, determined on a 2×16^3 lattice without any of the smearing mentioned in connection with the Ising action. They are *continuous* across the transition. We note that there is no apparent competition among the couplings. Further study of this S_{eff} is in progress.

Table 1

Terms in the effective 4-state Potts action and couplings at the phase transition on a 2×16^3 lattice

	operator \mathcal{O}^α	coupling β_α^*
single-spin	$\mathcal{O}^1 = \sum_{\mathbf{n}} \delta(s_{\mathbf{n}}, 0)$	-3.452
nn1	$\mathcal{O}^2 = \sum_{\mathbf{n}} \sum_{\mu} \delta(s_{\mathbf{n}}, s_{\mathbf{n}+\hat{\mu}})$	-0.638
nn2	$\mathcal{O}^3 = \sum_{\mathbf{n}} \sum_{\mu} \delta(s_{\mathbf{n}}, 0) \delta(s_{\mathbf{n}+\hat{\mu}}, 0)$	0.862
nnn1	$\mathcal{O}^4 = \sum_{\mathbf{n}} \sum_{\mu < \nu} \delta(s_{\mathbf{n}}, s_{\mathbf{n}+\hat{\mu} \pm \hat{\nu}})$	-0.104
nnn2	$\mathcal{O}^5 = \sum_{\mathbf{n}} \sum_{\mu < \nu} \delta(s_{\mathbf{n}}, 0) \delta(s_{\mathbf{n}+\hat{\mu} \pm \hat{\nu}}, 0)$	0.141
3 rd neighbor(1)	$\mathcal{O}^6 = \sum_{\mathbf{n}} \delta(s_{\mathbf{n}}, s_{\mathbf{n}+\hat{x} \pm \hat{y} \pm \hat{z}})$	-0.033
3 rd neighbor(2)	$\mathcal{O}^7 = \sum_{\mathbf{n}} \delta(s_{\mathbf{n}}, 0) \delta(s_{\mathbf{n}+\hat{x} \pm \hat{y} \pm \hat{z}}, 0)$	0.046
4 th neighbor(1)	$\mathcal{O}^8 = \sum_{\mathbf{n}} \sum_{\mu} \delta(s_{\mathbf{n}}, s_{\mathbf{n}+2\hat{\mu}})$	-0.049
4 th neighbor(2)	$\mathcal{O}^9 = \sum_{\mathbf{n}} \sum_{\mu} \delta(s_{\mathbf{n}}, 0) \delta(s_{\mathbf{n}+2\hat{\mu}}, 0)$	0.064

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