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Ising description of the transition region in SU(3) gauge theory at finite temperature

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Abstract

We attempt the numerical construction of an effective action in three dimensions for Ising spins which represent the Wilson lines in the four-dimensional SU(3) gauge theory at finite temperature. For each configuration of the gauge theory, each spin is determined by averaging the Wilson lines over a small neighborhood and then projecting the average to ± 1 according to whether the neighborhood is ordered or disordered. The effective Ising action, determined via the lattice Schwinger-Dyson equations, contains even (two-spin) and odd (one- and three-spin) terms with short range. We find that the truncation to Ising degrees of freedom produces an effective action which is discontinuous across the gauge theory's phase transition. This discontinuity may disappear if the effective action is made more elaborate.

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I. INTRODUCTION

In studying the fluctuations of a complex statistical field theory, it is frequently useful to define simple, effective degrees of freedom. A wise choice of these degrees of freedom allows one to focus on specific physics. We are interested in the first-order phase transition of the SU(3) gauge theory [1,2], specifically in surface phenomena such as the surface tension between the phases [3,4] and the stability of bubbles [5]. Since the transition is first-order, all correlation lengths stay finite on either side. This means that there should be no delicate issues related to renormalization-group fixed points and the symmetries that characterize them. We are thus led to degrees of freedom which simply specify whether a given region is in the confined or unconfined phase. Assigning values of ± 1 according to the two possibilities, we are led to an effective theory of Ising spins.

This exercise is familiar from study of the liquid–gas transition [6]. One assigns values to a local Ising spin σ according to the local density ρ of the fluid. At the first-order phase boundary $T = T^*$ the density is discontinuous, and this is reflected in a discontinuity in the magnetization $\langle\sigma\rangle$ of the effective Ising theory. The simplest Ising action has only one term that is odd in the spins, namely, the magnetic field term $h\sigma$. The first-order transition occurs perforce at $h = 0$, and one identifies the liquid–gas phase boundary with the segment of the T axis between the origin and the Ising critical point $T_{\rm cr}^{\rm Ising}$. As one slides along the phase boundary towards the liquid–gas critical point, the discontinuity in ρ decreases until it reaches zero at the critical point; correspondingly, the discontinuity in $\langle\sigma\rangle$ vanishes as one approaches $T_{\rm cr}^{\rm Ising}$.

The simplest Ising model, however, is incapable of describing an arbitrary first-order phase transition. An easy way to see this is to note that its $Z(2)$ symmetry implies that $\langle\sigma\rangle_{h\to 0^+} = -\langle\sigma\rangle_{h\to 0^-}$. Since the correspondence between σ and ρ depends on an arbitrary assignment in the first place, there is no reason to suppose that this symmetry reflects accurately the values of ρ on either side of T^*. Hence there must be more than one odd term in the Ising action, in order to move the transition away from $h = 0$ and thus to destroy the $Z(2)$ symmetry about the transition.

For the SU(3) confinement transition, we seek to characterize a neighborhood of a site $\mathbf{n}$ as confining or non-confining. We naturally settle on the Wilson line $L_{\mathbf{n}}$ as the quantity which does this. As an order parameter of the $Z(3)$ symmetry of the Euclidean theory, L is discontinuous at the transition, with $\langle L\rangle = 0$ in the confining phase below the transition and $\langle L\rangle \neq 0$ in the plasma phase above. We use the modulus of $L_{\mathbf{n}}$, suitably smeared, to assign a value to $\sigma_{\mathbf{n}} = \pm 1$. (This smearing reflects the fact that confinement is a property of a neighborhood, not of a point.) Running a Monte Carlo simulation of the gauge theory, we generate configurations of $L_{\mathbf{n}}$ which translate into configurations of $\sigma_{\mathbf{n}}$. We then use the Ising model's Schwinger-Dyson equations [7,8] to determine an approximation to the effective Ising action $S_{\rm eff}[\sigma]$.

This definition of σ as a function of L integrates over the $Z(3)$ dynamics of the Euclidean theory. Since we are interested in understanding bubbles of the confining phase in the plasma and vice-versa, distinguishing among the three ordered phases is unnecessary. Previous work [9] has taken the opposite approach, projecting the complex Wilson line onto 3-state Potts spins $\tau = \exp 2\pi n i/3$. This makes it harder to identify bubbles of the disordered phase because they will only be visible when calculating averages of τ over sizable neighborhoods.

Since our σ spins depend on the magnitude of L, they show directly those places where $|L|$ is small and hence disordered.

Since $Z(3)$ domain structure plays a role in the confinement phase transition, one might be concerned that domain walls should somehow be represented in the effective action. We note that calculations [4] have shown that, near the transition, the disordered phase wets the ordered phases, and moreover that two ordered domains with different $Z(3)$ orientations will sandwich a disordered domain between them. We note also that the physics of fluctuations among the three ordered phases will influence the Ising couplings because the effective action comes from an integration over *all* other degrees of freedom in the gauge theory. Nevertheless, integrating thus over the $Z(3)$ fluctuations may impair the ability of the Ising theory to represent the phase transition correctly. Our effective Ising action turns out to be discontinuous at the phase transition. Although this might be due to the small number of terms we permit in the action, it might stem from the use of Ising variables itself. We discuss this further below.

II. DEFINING THE EFFECTIVE ISING THEORY

We simulate the SU(3) gauge theory at finite temperature by using, as usual, a Euclidean lattice of $N_t \times N_s^3$ sites, with the physical temperature given by $T = (N_t a)^{-1}$ in terms of the lattice spacing a. All results presented in this paper were obtained from lattices with $N_t = 2$. The gauge theory is governed by the Wilson plaquette action,

$$S_W = \beta \sum_{\substack{n \\ \mu<\nu}} \mathrm{Tr}\, U_n^\mu U_{n+\hat{\mu}}^\nu U_{n+\hat{\nu}}^{\mu\dagger} U_n^{\nu\dagger} \ , \tag{1}$$

with $U_n^\mu \in \mathrm{SU}(3)$. The order parameter for the confinement phase transition is the Wilson line, defined on a site $\mathbf{n}$ of a three-dimensional lattice via

$$L_{\mathbf{n}} = \mathrm{Tr} \prod_{n_0=1}^{N_t} U^0_{(\mathbf{n},n_0)} \ . \tag{2}$$

As discussed in the Introduction, we will use L to define the effective Ising spins σ which will label a neighborhood on the lattice as confining or non-confining. A first attempt might be to define

$$\sigma_{\mathbf{n}} = \begin{cases} -1, & |L_{\mathbf{n}}| < r_\sigma \\ 1, & |L_{\mathbf{n}}| > r_\sigma \end{cases} \tag{3}$$

The problem with this is that $L_{\mathbf{n}}$ fluctuates violently from site to site. Even deep in the connfining phase, where $\langle L \rangle = 0$, $L_{\mathbf{n}}$ is by no means confined to a region around zero, and in fact fills the entire wedge available to it in the complex plane (see Fig. 1). σ as defined by (3) thus does not offer a good definition of a domain in the confining phase.

The fluctuations in $L_{\mathbf{n}}$ are reduced if it is averaged over a small volume. We define $L_{\mathbf{n}}^{[m^3]}$ to be the average of L over the $m \times m \times m$ block surrounding[1] $\mathbf{n}$. A glance at Fig. 1

[1] If m is even then $\mathbf{n}$ is a site of the dual lattice.

shows that $L^{[8]}_{\mathbf{n}}$ discriminates well, on a local basis, between domains that resemble the two respective phases. $L^{[27]}_{\mathbf{n}}$, on the other hand, fluctuates too little about the volume average of L, so that, with a reasonable value chosen for r_σ, the σ spins would lose all information about fluctuations and remain entirely ordered with $\sigma = \pm 1$. We thus choose $L^{[8]}_{\mathbf{n}}$ for insertion in (3) to calculate the σ configurations; as shown in Fig. 2, we set $r_\sigma^2 = 0.8$.

With the definition of σ in hand, we turn to the determination of the effective Ising action. In principle the action has an infinite number of terms; we truncate it to a combination of a magnetic field term and two- and three- spin terms with range 2,

$$S_{\text{eff}}[\sigma] = \sum_\alpha \beta_\alpha \mathcal{O}^\alpha \ , \tag{4}$$

where the seven operators $\mathcal{O}^\alpha$ are listed in Table I. The two-spin operators $\mathcal{O}^2$ and $\mathcal{O}^3$, as well as the three-spin operator $\mathcal{O}^4$, couple spins within distance $\sqrt{2}$; the remaining operators reach out to distance 2.

A Schwinger-Dyson equation of the Ising theory is derived by flipping a spin $\sigma_{\mathbf{n}}$ in the sum defining the expectation value of some operator. For the operators in Table I, we have

$$\left\langle \tilde{\mathcal{O}}^\alpha_{\mathbf{n}} \right\rangle = -\left\langle \tilde{\mathcal{O}}^\alpha_{\mathbf{n}} \exp 2\tilde{\mathcal{S}}_{\mathbf{n}} \right\rangle \tag{5}$$

where we have defined $\tilde{\mathcal{O}}^\alpha_{\mathbf{n}}$ to be those terms in $\mathcal{O}^\alpha$ that contain $\sigma_{\mathbf{n}}$, and

$$\tilde{\mathcal{S}}_{\mathbf{n}} = \sum_\alpha \beta_\alpha \tilde{\mathcal{O}}^\alpha_{\mathbf{n}} \tag{6}$$

is the part of the action that contains $\sigma_{\mathbf{n}}$. These are seven equations for determining the seven unknowns β_α. After generating an ensemble of σ configurations via Monte Carlo simulation of the gauge theory, we determine β_α iteratively as solutions of (5).

As a consistency check, one may use the vacuum equation

$$1 = \left\langle \exp 2\tilde{\mathcal{S}}_{\mathbf{n}} \right\rangle \tag{7}$$

or the Schwinger-Dyson equation for any other operator in the theory. A more satisfying check, however, is to run a direct Monte Carlo simulation of the Ising model with action (4) to see if the expectation values of $\mathcal{O}^\alpha$ as computed in the gauge theory are reproduced. This was the procedure we followed. We calculated error estimates by subdividing the ensemble.

III. RESULTS AND DISCUSSION

We simulated the SU(3) gauge theory on a lattice of volume 2×16^3. The confinement phase transition is in the neighborhood [10] of $\beta = 5.09$, and we settled on the value $\beta = 5.091$ after seeing no tunneling between the coexisting phases in moderately long runs at that coupling.

Straightforward application of the method described above gives an Ising action for a three-dimensional lattice of volume 16^3. We show the couplings for this action, derived from ordered and disordered runs at $\beta = 5.091$, in Table II. In both cases, the action contains couplings with range $\sqrt{3}$ and 2 ($\mathcal{O}^6$ and $\mathcal{O}^7$, respectively) which are as strong as the shorter-ranged two-spin couplings ($\mathcal{O}^2$ and $\mathcal{O}^3$) and compete with them in sign.[2] This raises the suspicion that a longer-ranged Ising action is needed to reproduce the Ising configurations correctly, and that the range-2 action is too crude a truncation. This suspicion is confirmed by simulating directly the Ising model with the couplings just derived. As seen in Table III, comparison of $\langle\mathcal{O}^\alpha\rangle$ with the averages from the gauge configurations shows poor agreement.[3]

This problem was encountered by Deckert *et al.* [11] in a calculation of the effective action for the $Z(2)$ gauge theory. A solution, noted in [8], is to perform a block-spin transformation on the spins, so that the effective action has twice the range. We do this simply by decimating the Ising spins, already defined via smeared averages, to an 8^3 sublattice. Solving the Schwinger-Dyson equations with the decimated configurations gives the couplings shown in Table IV. The longer-ranged two-spin couplings, β_6 and β_7, are negligible, as is the straight three-spin coupling β_5. Moreover, comparison of an Ising Monte Carlo simulation with the gauge theory (see Table V) now gives satisfactory agreement.

The effective Ising couplings shown in Table IV vary smoothly as β is varied on either side of the transition, but they are discontinuous across the transition. At $\beta = 5.091$ we have, then, *two* actions, S_{cold} and S_{hot}, which are the limits of $S_{\text{eff}}[\sigma]$ from the ordered and disordered sides of the transition. Curiously, we find that S_{hot} is at a point of phase coexistence, that is, at a phase transition between phases with $\langle\sigma\rangle < 0$ and $\langle\sigma\rangle > 0$. We show in Table V the expectation values of the seven operators $\mathcal{O}^\alpha$ for S_{cold} and for both phases of S_{hot}. For S_{hot}, the phase with $\langle\sigma\rangle < 0$ describes well the expectation values in the gauge theory on the disordered side of its transition. The other phase of S_{hot}, of course, does not; neither does it describe the ordered phase of the gauge theory. The action S_{hot} thus "knows" that it describes a phase transition, but it is capable of describing correctly only one of the phases. S_{cold} describes the ordered phase well, and shows no phase coexistence.

The discontinuity in the effective action is an example of the singularities that can result from renormalization group transformations. Griffiths and Pearce [12] noted that a blocked action might be a singular function of the unblocked couplings even though the blocking transformation is local. Later work [13] found discontinuities in the blocked action associated with first-order phase transitions in the original action. It was conjectured that there may be different renormalization-group flows resulting from the various metastable phases at a fixed coupling. In view of theorems proven by van Enter, Fernández, and Sokal [14], however, such discontinuities are impossible in an effective action which possesses finite range in the infinite-volume limit. Our effective action, however, is approximate in that it contains a

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[2]Note that the magnetic field is $h = -\beta_1$, and that *negative* values for β_2, β_3, β_6, and β_7 indicate ferromagnetic couplings.

[3]The violent disagreement for the ordered phase, including even the sign of the magnetization, suggests that the gauge theory's operator averages are to be sought in a metastable phase of the Ising action. We did not succeed, however, in reaching this phase with our Monte Carlo. In the disordered phase, the positive magnetic field $h = -\beta_1$ prefers a positive magnetization, but the positive three-spin couplings β_4 and β_5 compete with it and turn the magnetization negative.

small number of couplings. Adding longer-ranged and multi-spin terms to the action will bring consistency with the theorem of van Enter *et al.* in one of two ways: Either the couplings will become continuous [15], or the effective action(s) will acquire too many non-local terms in the infinite-volume limit, meaning the statistical measure is non-Gibbsian. In the first case, we will have an effective action well suited to describing the phase transition; in the second case, the conclusion will be that an Ising description of the phase transition is impossible. Deciding between these alternatives requires great numerical precision.

The discontinuity of the effective action is sensitive to the definition of the effective degrees of freedom, just as singularities in the renormalization group may be created or eliminated by different choices of the block-spin transformation. A more sophisticated definition of $\sigma_{\mathbf{n}}$, perhaps using a Kadanoff kernel to associate it with the smeared $L_{\mathbf{n}}$, may restore continuity, even without marked increase in the number of interaction terms. Note also that a reduction of the gauge theory to $Z(3)$ spins in [9] resulted in an action that is continuous across the phase transition. Perhaps a more complex effective spin, combining Ising with $Z(3)$, will yield an effective action that offers both continuity and a local description of confinement physics.

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FIGURES

FIG. 1. Distribution of the Wilson line $L_{\mathbf{n}}$, averaged over $m \times m \times m$ cubes, in the complex plane. Left: $\beta = 5.0$ (disordered phase). Right: $\beta = 5.2$ (ordered phase). Top to bottom: $m = 1$ (single site), $m = 2$, $m = 3$. The lattice size is 2×8^3.

FIG. 2. $L_{\mathbf{n}}$ distributions for $m = 2$, as in Fig. 1, with circle $|L_{\mathbf{n}}|^2 = r_\sigma^2 = 0.8$ superimposed.

TABLES

TABLE I. Operators $\mathcal{O}^\alpha$ appearing in the truncated effective Ising action.

single spin	$\mathcal{O}^1 = \sum_{\mathbf{n}} \sigma_{\mathbf{n}}$
nearest neighbor	$\mathcal{O}^2 = \sum_{\mathbf{n}} \sum_{\mu} \sigma_{\mathbf{n}} \sigma_{\mathbf{n}+\hat{\mu}}$
next-nearest neighbor	$\mathcal{O}^3 = \sum_{\mathbf{n}} \sum_{\mu<\nu} \sigma_{\mathbf{n}} \sigma_{\mathbf{n}+\hat{\mu}\pm\hat{\nu}}$
3-spin bent	$\mathcal{O}^4 = \sum_{\mathbf{n}} \sum_{\mu<\nu} \sigma_{\mathbf{n}} \sigma_{\mathbf{n}\pm\hat{\mu}} \sigma_{\mathbf{n}\pm\hat{\nu}}$
3-spin straight	$\mathcal{O}^5 = \sum_{\mathbf{n}} \sum_{\mu} \sigma_{\mathbf{n}} \sigma_{\mathbf{n}-\hat{\mu}} \sigma_{\mathbf{n}+\hat{\mu}}$
$3^{\rm rd}$ neighbor	$\mathcal{O}^6 = \sum_{\mathbf{n}} \sigma_{\mathbf{n}} \sigma_{\mathbf{n}+\hat{x}\pm\hat{y}\pm\hat{z}}$
$4^{\rm th}$ neighbor	$\mathcal{O}^7 = \sum_{\mathbf{n}} \sum_{\mu} \sigma_{\mathbf{n}} \sigma_{\mathbf{n}+2\hat{\mu}}$

TABLE II. Couplings β_α for the (tentative) effective Ising action on a 16^3 lattice, for the ordered and disordered phases at $\beta = 5.091$.

α	ordered	disordered
1	0.054(4)	-0.135(16)
2	-0.455(1)	-0.390(8)
3	-0.052(1)	-0.026(2)
4	-0.0056(6)	0.021(1)
5	0.0063(9)	0.022(2)
6	0.044(1)	0.045(4)
7	0.152(1)	0.132(3)

TABLE III. Averages of $\mathcal{O}^\alpha$ in ordered and disordered phases of the gauge theory at $\beta = 5.091$, compared with results of Ising Monte Carlo for the couplings listed in Table II. Averages are normalized to 1.

α	ordered phase		disordered phase	
	gauge theory	Ising MC	gauge theory	Ising MC
1	0.185(7)	-0.849(2)	-0.877(1)	-0.696(6)
2	0.500(1)	0.812(2)	0.837(1)	0.708(4)
3	0.353(1)	0.775(2)	0.804(1)	0.626(5)
4	0.137(6)	-0.738(3)	-0.779(1)	-0.595(6)
5	0.141(6)	-0.742(2)	-0.781(1)	-0.602(6)
6	0.269(1)	0.756(3)	0.789(1)	0.583(6)
7	0.206(2)	0.743(3)	0.781(1)	0.552(7)

TABLE IV. Couplings β_α for the effective Ising action after decimation to an 8^3 lattice, for several gauge couplings surrounding the phase transition.

α	$\beta = 5.08$	$\beta = 5.091$ (disordered)	$\beta = 5.091$ (ordered)	$\beta = 5.1$	$\beta = 5.2$
1	0.14(6)	0.02(5)	-0.027(2)	-0.054(2)	-0.26(3)
2	-0.21(2)	-0.24(2)	-0.132(1)	-0.131(1)	-0.12(1)
3	-0.043(3)	-0.050(3)	-0.023(1)	-0.021(1)	-0.025(3)
4	-0.011(2)	-0.015(2)	0.0043(4)	0.0028(4)	-0.0014(16)
5	0.003(5)	-0.001(7)	-0.003(2)	-0.0003(9)	-0.004(4)
6	-0.002(3)	-0.002(4)	-0.006(1)	-0.005(1)	-0.006(2)
7	-0.003(2)	0.003(7)	0.0016(7)	0.005(1)	0.004(3)

TABLE V. Comparison of operator averages $\langle\mathcal{O}^\alpha\rangle$ in ordered and disordered phases of the gauge theory at $\beta = 5.091$, compared with the effective Ising actions $S_{\rm cold}$ and $S_{\rm hot}$ simulated directly.

α	gauge theory (ordered)	$S_{\rm cold}$	$S_{\rm hot}$		gauge theory (disordered)
			$\langle\sigma\rangle > 0$	$\langle\sigma\rangle < 0$	
1	0.188(7)	0.154(2)	0.9866(1)	-0.8799(1)	-0.878(1)
2	0.206(2)	0.203(1)	0.9739(1)	0.7849(2)	0.782(2)
3	0.123(2)	0.119(1)	0.9734(1)	0.7772(2)	0.774(2)
4	0.060(3)	0.049(1)	0.9615(1)	-0.7006(2)	-0.698(2)
5	0.063(3)	0.053(1)	0.9614(1)	-0.6994(2)	-0.697(2)
6	0.092(2)	0.086(1)	0.9733(1)	0.7751(1)	0.772(2)
7	0.080(2)	0.073(1)	0.9733(1)	0.7745(2)	0.772(2)

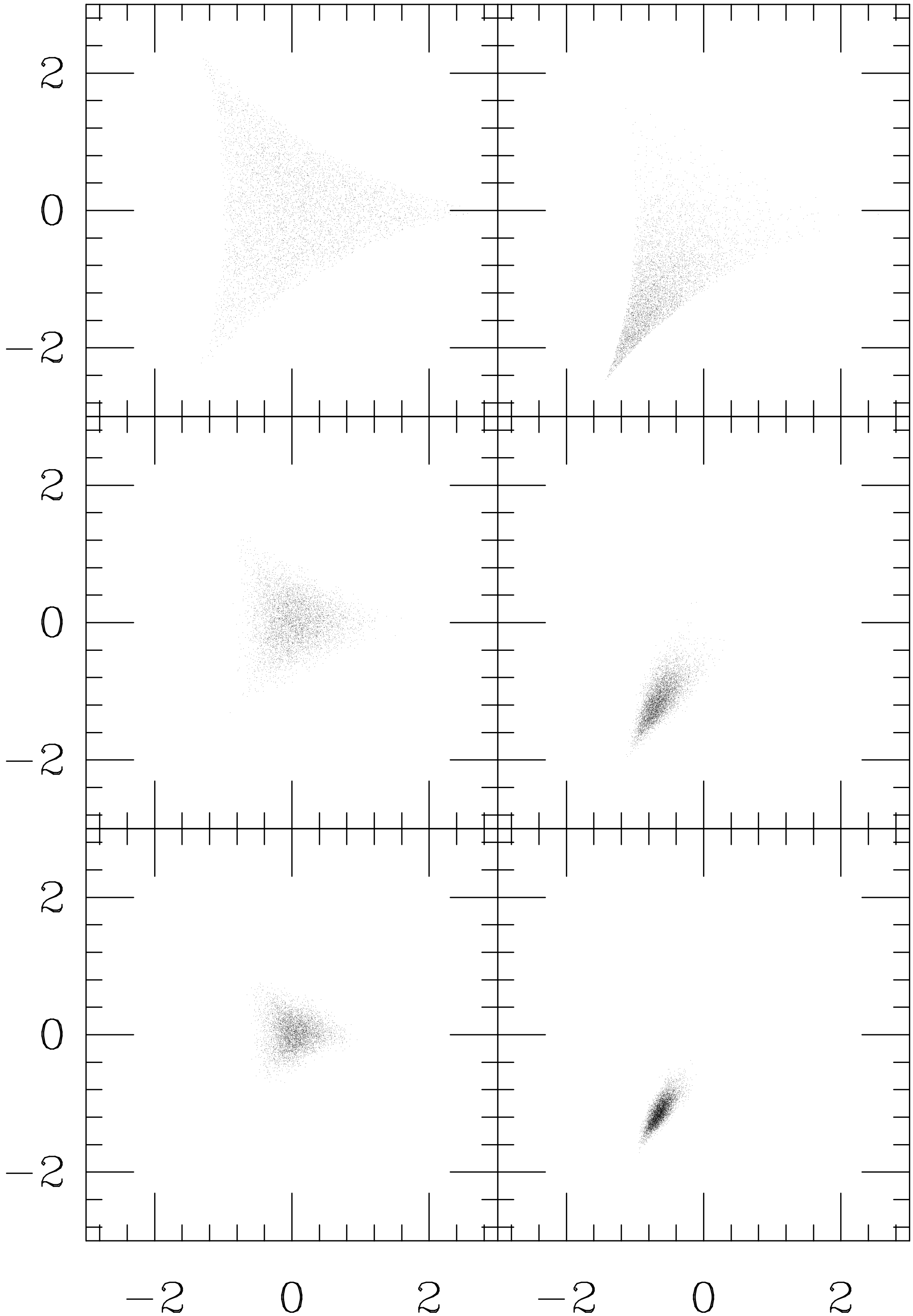

FIG. 1

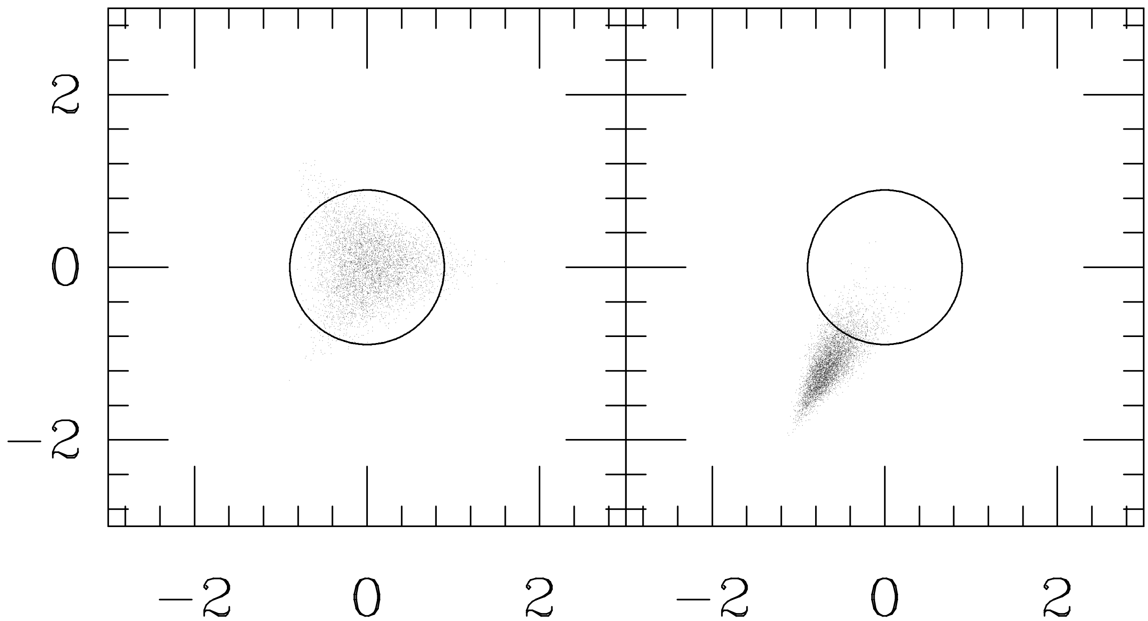

FIG. 2