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INSTABILITY OF BUBBLES IN THE QUARK-GLUON PLASMA

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ABSTRACT. The small surface tension of vacuum bubbles in the quark-gluon plasma, along with a negative curvature tension, implies that the uniform plasma is unstable against the formation of such bubbles. We show that spherical bubbles are in turn unstable against the growth of irregularities.

Farhi and Jaffe [1] noted some time ago that the MIT bag model gives an anomalously small surface tension between the high- and low-temperature phases of QCD. Consider a spherical bubble of the vacuum phase embedded in the bulk plasma. One defines this surface tension σ via an expansion of the bubble's free energy in inverse powers of its radius R,

$$F(R) = \Delta P\, V + \sigma A + 8\pi\alpha R + \cdots \ , \tag{1}$$

where V and A are respectively the volume and surface area of the bubble, and ΔP is the difference between the pressures of the two phases (zero at the transition temperature $T = T_0$, and positive above); we have defined as well the "curvature coefficient" α. In bag model terms, the surface tension is considered small when $\sigma \ll B^{3/4}$, since the bag constant B determines all the bulk physics, including the transition temperature and latent heat. The smallness of σ has been confirmed in lattice gauge theory calculations for the pure glue theory [2].

If σ is small, one is led to consider the value and importance of the curvature term, and perhaps of higher orders in $1/R$ in (1). Mardor and Svetitsky [3] found, again in the bag model, that α is sizeable and negative. This implies that vacuum bubbles are *stable*, and it was shown in [3] that this stability also emerges from an exact bag model calculation, not presuming the validity of the expansion (1). It was further suggested [3,4] that this result might transcend the bag model: All that is required is a thin interface, the energy of which is due primarily to the reaction of the fields on either side. A lattice calculation of α in the pure glue theory [5] apparently supports this picture, even though the interface does *not* turn out to be thin.

The fact that one can lower the free energy of the plasma phase by creating a vacuum bubble seems at first glance to imply that the equilibrium plasma is a Swiss cheese of bubbles, the density of which is determined by the (unknown) bubble–bubble interaction. This would only be true, however, if (1) the bubbles do not affect each other, and (2) each bubble is stable in itself. We present in this note some simple calculations which show that spherical bubbles are unstable against deformation, and that growth of long, narrow structures is favored. We do not address the problem of determining the actual equilibrium configuration of the high-temperature phase, a problem which will require much more sophisticated analysis.

In order to study the stability of the spherical bubble against small perturbations, we expand the free energy around the equilibrium spherical bubble, of radius R_0. Minimizing (1), we find that R_0 satisfies

$$4\pi\Delta P\, R_0^2 + 8\pi\sigma R_0 + 8\pi\alpha = 0\ . \tag{2}$$

The generalization of (1) to non-spherical shapes is [6]

$$F = \Delta P\, V + \sigma\int dS + \alpha\int dS\,\left(\frac{1}{R_1}+\frac{1}{R_2}\right) + \cdots\ , \tag{3}$$

where the integrand in the third term is the local extrinsic curvature, with R_1 and R_2 representing the principal radii of curvature. Let the bubble surface be specified by

$$R(\theta,\phi) = R_0 + \delta R(\theta,\phi)\quad . \tag{4}$$

We demand that δR have continuous derivatives and satisfy $\delta R \ll R_0$. To second order in δR, the change in the curvature integral in (3) is

$$\int\left[2\,\delta R\sin\theta - \frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\delta R\right) - \frac{1}{\sin\theta}\frac{\partial^2}{\partial\phi^2}\delta R\right]d\theta\,d\phi \\ + \frac{1}{R_0}\int\left[\sin\theta\left(\frac{\partial}{\partial\theta}\delta R\right)^2 + \frac{1}{\sin\theta}\left(\frac{\partial}{\partial\phi}\delta R\right)^2\right]d\theta\,d\phi\ . \tag{5}$$

The last two terms in the $O(\delta R)$ contribution vanish on integration.

The total $O(\delta R)$ contribution to F is

$$\delta F = \left(\Delta P\, R_0^2 + 2\sigma R_0 + 2\alpha\right) \int \sin\theta\, d\theta\, d\phi\, \delta R \tag{6}$$

which is identically zero when R_0 is the equilibrium radius, since the coefficient is proportional to (2). The correction to F is thus second order in δR,

$$\delta F = \int \sin\theta\, d\theta\, d\phi \left\{ A\,(\delta R)^2 + B\left[\left(\frac{\partial}{\partial\theta}\delta R\right)^2 + \frac{1}{\sin^2\theta}\left(\frac{\partial}{\partial\phi}\delta R\right)^2\right]\right\} , \tag{7}$$

where

$$A = \Delta P\, R_0 + \sigma\ , \qquad B = \frac{\sigma}{2} + \frac{\alpha}{R_0} \quad . \tag{8}$$

Recall that we presume $\alpha < 0$. When R_0 is determined by (2), it is easy to see that A is positive and B negative. Integrating the second term in (7) by parts, we obtain

$$\begin{aligned} \delta F &= \int \sin\theta\, d\theta\, d\phi\, \delta R \left\{ A\,\delta R - B\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\delta R\right) + \frac{1}{\sin^2\theta}\left(\frac{\partial^2}{\partial\phi^2}\delta R\right)\right]\right\} \\ &= \int \sin\theta\, d\theta\, d\phi\, \delta R\left(A + BL^2\right)\delta R\ . \end{aligned} \tag{9}$$

$-L^2$ is just the angular part of the Laplacian operator. In fact, from symmetry considerations, (9) is the only possible form that could have emerged, and the calculation was needed only to determine the coefficient B.

Expanding δR in a multipole expansion,

$$\delta R(\theta,\phi) = \sum_{l,m} c_l^m Y_{lm}(\theta,\phi)\ , \tag{10}$$

we have

$$\delta F = \sum_{l,m}[A + Bl(l+1)]\,|c_l^m|^2 \quad . \tag{11}$$

Evidently $\delta F > 0$ for a monopole perturbation, since (1) has been minimized; for a dipole perturbation we obtain $\delta F = 0$ [see (2)], consistent with the fact that $l = 1$ corresponds to a translation of the bubble. For quadrupole and higher deformations, however, δF is always negative, showing instability against arbitrary changes of shape.

One particular deformation is the growth of a finger. Consider pushing a polar cap of the sphere outward, so that $\delta R(\theta) = a$ for $\theta \lesssim \theta_0 \ll 1$, with the transition region from $\delta R = 0$ to $\delta R = a$ having a thickness $\delta\theta \ll \theta_0$. A rough estimate of (7) gives

$$\delta F \sim A\pi R_0^2\theta_0^2 a^2 + B2\pi R_0\theta_0\frac{a^2}{(\delta\theta)^2}\ . \tag{12}$$

δF can be made negative by making $\delta\theta$ sufficiently small, whereupon the overall proportionality to a^2 makes the finger grow ever longer.

To see the eventual fate of such a finger, we must depart from the expansion of F in δR. Since the finger appears to grow into a cylinder, let us calculate the free energy of a cylinder of radius R and length L. Neglecting the ends, we find that the volume, area, and surface terms contribute

$$F = \Delta P \pi R^2 L + \sigma 2\pi R L + \alpha 2\pi L \ . \tag{13}$$

When the coefficient of L is positive, the cylinder tends to shrink in both the radial and axial directions. However, when R shrinks past $R_{\rm crit}$, which satisfies

$$\frac{\Delta P}{2} R_{\rm crit}^2 + \sigma R_{\rm crit} + \alpha = 0 \ , \tag{14}$$

the coefficient of L becomes negative, and the cylinder lowers its free energy by growing longer. Note that for $R < R_{\rm crit}$ the overall free energy of the cylinder is negative, and thus the uniform plasma is unstable against nucleation of such cylinders.

The validity of our analysis is restricted in two respects. Firstly, we have not taken into account the finite thickness of the interface, which is estimated [2,5] to be approximately $2\,T_0^{-1}$. Secondly, the $1/R$ expansion (1) may be reliably truncated only as long as R is much larger than the ratio of successive coefficients in the series; this ratio in turn must be related to the dynamical scales in the system. The two dynamical scales are given by the bag constant (or some other fundamental scale of QCD) and the temperature; near the transition, the two are comparable.

Near T_0, where ΔP may be neglected, the solution of (2) is $R_0 = -\alpha/\sigma \simeq 18\,T_0^{-1}$, where we have used the values [3,5] $\alpha \simeq -4T_0^2/9$ and [2] $\sigma \simeq 0.024\,T_0^3$. This evidently satisfies the constraints of validity.

Farther from T_0, where the pressure term in (2) overwhelms the surface tension term, we estimate $R_0 = \sqrt{2\alpha/\Delta P}$. The bag model gives

$$\Delta P = \frac{\pi^2}{90}(\gamma_q - \gamma_h)(T^4 - T_0^4) \ , \tag{15}$$

where $\gamma_q = 37$ and $\gamma_h = 3$ are the effective degeneracy factors stemming from counting the light degrees of freedom in the quark and hadron phases. Near T_0, we write $\Delta P \simeq 16\,T_0^3(T - T_0)$; ignoring σ, this gives $R_0 \simeq \left[6\sqrt{T_0(T-T_0)}\right]^{-1}$. Demanding $R_0 > 2T_0^{-1}$ gives $T - T_0 < 0.01\,T_0$. If one were to eschew the bag formula for ΔP in favor of a lattice calculation, one would find a substantially smaller ΔP near the transition, since $\Delta P \sim \Delta\epsilon(T - T_0)$, where $\Delta\epsilon$ is the latent heat; lattice calculations give a much smaller latent heat than does the bag model. Thus the temperature range where bubbles are large may be significantly larger than the above estimate.

We have here considered only static properties of the quark–hadron interface. Dynamical processes in the course of the phase transition will destabilize the interface even more. For example, the necessity of carrying heat and baryon number away from a growing vacuum bubble will lead to a negative effective surface tension and hence to complex phenomena of pattern formation [7].

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REFERENCES

1. E. Farhi and R. L. Jaffe, Phys. Rev. D **30** (1984) 2379.
2. K. Kajantie and L. Kärkkäinen, Phys. Lett. B **214** (1988) 595;
   K. Kajantie, L. Kärkkäinen, and K. Rummukainen, Nucl. Phys. **B333** (1990) 100.
   S. Huang, J. Potvin, C. Rebbi, and S. Sanielevici, Phys. Rev. D **42** (1990) 2864; **43** (1991) 2056(E).
   R. Brower, S. Huang, J. Potvin, and C. Rebbi, Boston University preprint BUHEP-92-3 (unpublished, 1992).
3. I. Mardor and B. Svetitsky, Phys. Rev. D **44** (1991) 878.
4. B. Svetitsky, lecture given at the Workshop on QCD at Finite Temperature and Density, Brookhaven National Laboratory, August 1991, Tel Aviv preprint TAUP 1909–91 (1991), to appear in proceedings.
5. K. Kajantie, L. Kärkkäinen, and K. Rummukainen, Helsinki preprint HU-TFT-92-1 (unpublished, 1992).
6. R. Balian and C. Bloch, Ann. Phys. (NY) **60** (1970) 401.
7. J. S. Langer, Rev. Mod. Phys **52** (1980) 1.