STATUS OF LATTICE GAUGE THEORY*

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I review recent results in lattice gauge theory which pertain to the physics of high temperature QCD and quark matter. Topics covered include: the transition temperature in the pure gauge theory, scaling and finite temperature behavior in QCD with quarks, problems with non-zero chemical potential, the Debye length in the high temperature plasma, and quantities relevant to thermodynamics and hydrodynamics in relativistic heavy ion collisions.

1. PREFACE

Two years ago in Helsinki, Tom DeGrand summarized the state of the art in lattice gauge theory[1]. He gave tantalizing hints of what the future may bring, and exhorted lattice theorists to work harder. I will present a picture of lattice gauge theory today which will show that theorists have indeed been working hard, and that the future has begun to arrive. Ambitious research programs are starting to give answers to some of the questions we care about the most, and much of the early confusion in lattice results is yielding to systematic analysis. I will be forced, however, to temper my boastful optimism with some bitter realism, and warn you that further progress with current algorithms will require new generations of supercomputers.

I will discuss only results relevant to the physics of quark matter, and I will concentrate on progress since Helsinki. First I will deal with a nagging loose end in the study of pure gauge theory (quarkless QCD), namely, what is the transition temperature $T_{\rm tr}$? The problem here has been the general one of taking the continuum limit of lattice results. I will describe the essence of the problem, and the results of recent calculations.

Next I will summarize the rather startling progress that has been accomplished in simulations of full QCD, quarks and all. Most surprising is the preliminary evidence that continuum scaling is being seen on the rather small lattices accessible to current calculations. I will show Monte Carlo data for the finite-temperature chiral symmetry phase transition temperature; if the tentative scaling behavior is the real thing, then the calculated value of the transition temperature may not be totally fanciful. The phenomenological interest in this transition is of course obvious; equally interesting would be information about the phase structure of QCD at non-zero chemical potential (i.e., non-zero baryon density). I will speak briefly of the formal problems which prevent analysis of this regime.

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While the numerical studies of QCD are glamorous and impressive, they are close to yielding information only about a bare minimum of physically interesting quantities. The phenomenology of relativistic heavy ion collisions will, on the other hand, require rather detailed predictions. To begin with, the high-temperature quark-gluon plasma is now believed to be considerably more complex than its electrodynamic counterpart. I will describe some of the conundrums the theory poses, and show some Monte Carlo data (in the pure gauge theory) which address them. Finally, I will give publicity to crude but audacious attempts to calculate quantities relevant to the phenomenological hydrodynamical theories of high energy collisions.

2. LATTICE GAUGE THEORY IN A NUTSHELL

The conferences in this series have benefitted from excellent review talks on lattice gauge theory by Helmut Satz[2], John Kogut[3], and Tom DeGrand[1]. I presume, however, that there are some people in the audience who have successfully resisted being turned into experts in the field. Thus I begin with a brief description of what the lattice approach is and what its basic problems are[4].

Much of quantum field theory can be boiled down to the problem of how to deal with the Feynman path integral formula for the thermodynamic partition function,

$$Z = \int [dA_\mu][d\psi][d\bar\psi] \, \exp\left[-S(A_\mu,\psi,\bar\psi)\right] \;, \tag{1}$$

where (in the case of QCD) S is the Euclidean action written in terms of the glue field A_μ and the quark fields ψ and $\bar\psi$. The fields are defined in a funny space-time, the temporal extent of which is finite and given by the inverse temperature $\beta \equiv 1/T$, and periodic boundary conditions are imposed at $t=0$ and $t=\beta$. The partition function is the integral over all space-time configurations of the fields.

The first step, necessary for *any* calculation in quantum field theory, is to impose a short distance cutoff so that intermediate results of calculations will be finite. We choose to do this via the introduction of a space-time lattice with $N_t \times N_s^3$ sites and lattice spacing a: the fields now live on the sites and links of the lattice, and the inverse temperature is given by the temporal extent of the lattice via $\beta = N_t a$.

The next step—the hardest—is to evaluate expectation values of operators in the thermodynamic ensemble. The general definition is

$$\langle\mathcal{O}\rangle \equiv Z^{-1} \int [dA_\mu][d\psi][d\bar\psi] \, e^{-S}\, \mathcal{O}, \tag{2}$$

where $\mathcal{O}$ is some function of the field variables. Eschewing perturbation theory and other analytic approximations, let us contemplate a direct numerical calculation. I will not dwell on how many variables there are to integrate over in (1) and (2), or on how many times the age of the universe it would take to do those integrals on a computer. Suffice it to say that even if we could do it in only, say, a tenth the age of the universe, we would probably

FIGURE 1
The space-time lattice

be too late to make concrete predictions before RHIC comes on line. Thus one is forced to adopt a Monte Carlo strategy. We store in the computer's memory a single space-time configuration of the fields, and we give the computer a rule for generating another configuration from it. This rule is rigged so that the sequence of configurations generated turns out to be distributed (in the long run) as e^{-S}. We calculate the value of $\mathcal{O}$ in each configuration, and we estimate the expectation value $\langle\mathcal{O}\rangle$ by averaging the values of $\mathcal{O}$ in the sequence.

The only input parameter to the computer calculation is the dimensionless coupling constant g. Simple dimensional analysis shows that $\langle\mathcal{O}\rangle$ is necessarily of the form

$$\langle\mathcal{O}\rangle = a^{-d} f(g), \tag{3}$$

where d is the dimension of $\mathcal{O}$ (in mass units) and $f(g)$ is the (dimensionless) function of g given by the computer output. The final step in deriving a physical result is to ask the obvious question, what is the lattice spacing a? In the end, of course, a must go to zero, giving the continuum limit and the physics of the continuum theory via renormalization. An operational prescription for extracting this physics is the following. a is a function of the coupling, taking the form

$$a = \Lambda_L^{-1} R(g), \tag{4}$$

where Λ_L is an arbitrary scale with dimensions of mass and R is a dimensionless function. R must have the property that the $a \to 0$ limit is the same as $g \to 0$, in accordance with asymptotic freedom (since g is the effective coupling at the scale of the lattice spacing). When g is small, perturbation theory may be used to derive the expression

$$R(g) = \left(\frac{11g^2}{16\pi^2}\right) \exp\left(-\frac{8\pi^2}{11g^2}\right) \tag{5}$$

for the pure gauge theory. (The coefficients depend on the number of quark flavors.) Inserting (4) into (3) gives

$$\langle\mathcal{O}\rangle = \Lambda_L^d \, R(g)^{-d} \, f(g). \tag{6}$$

Thus if $\langle \mathcal{O} \rangle$ is to have a finite limit then $f(g)$ had better behave as $f \sim R^d$ as $g \to 0$. The final result of the calculation is then that $\langle \mathcal{O} \rangle$ takes the value f/R^d in units of the lattice scale Λ_L. Note that $R(g)$ is a *universal* function, not dependent on the observable being calculated; $f(g)$ is of course different for each observable.

Now consider the temperature T. It is given by $T = [N_t a(g)]^{-1} = \Lambda_L [N_t R(g)]^{-1}$. Maintaining a finite non-zero temperature in the limit $a \to 0$ means that $N_t \to \infty$: calculations near the continuum limit demand ever larger lattices. The *transition* temperature T_{tr} is given by $T_{\mathrm{tr}} = [N_t a(g_{\mathrm{tr}})]^{-1}$, where g_{tr} is the coupling at which the transition occurs on a lattice of fixed size (it varies with N_t). Using (4),

$$T_{\mathrm{tr}} = \Lambda_L [N_t R(g_{\mathrm{tr}})]^{-1}, \tag{7}$$

and this combination should approach a finite limit as g (and a) approach zero.

3. THE TRANSITION TEMPERATURE IN THE PURE GAUGE THEORY

Now we can discuss recent progress in calculating T_{tr} in the pure gauge theory as an exercise in taking the continuum limit. According to (7), one calculates g_{tr} on lattices with various values of N_t, and then uses (when g is sufficiently small) the formula (5) for the effective lattice spacing to obtain the dimensionless ratio $T_{\mathrm{tr}}/\Lambda_L$. Recent Monte Carlo results[5] for this ratio are shown in Fig. 2. The points to the right of $6/g^2 = 6.0$ show no drift as the continuum ($g \to 0$) limit is approached. In years past, only points to the left of $6/g^2 = 5.7$ were available, and it was thought[6] that the ratio had settled down to a continuum limit in the neighborhood of $T_{\mathrm{tr}}/\Lambda_L \sim 76$. It is hoped that the move to the new value is a final one. (A competing group[7] agrees that $T_{\mathrm{tr}}/\Lambda_L$ is constant for $6/g^2 > 6.0$, but reports a value 10% lower than that shown in Fig. 2. The discrepancy is due to systematics.)

The lattice scale Λ_L may not look familiar, but the ratio $\Lambda_L/\Lambda_{\overline{MS}}$ is calculable in perturbation theory[8], so that the scale of the theory may be related to that of the real world. Unfortunately, $\Lambda_{\overline{MS}}$ in the real world pertains to *full* QCD, not to a quarkless version; moreover, it is only known to within a factor of 2. Thus it has become customary to relate results in the pure gauge theory to the string tension σ. The string tension is calculated at $T = 0$, and the formula (3) takes the form

$$\sigma = a^{-2} f_\sigma(g), \tag{8}$$

where f_σ is the output of the computer calculation. Equation (4) allows us to define the dimensionless ratio σ/Λ_L^2, which should approach a finite continuum limit. This behavior is seen in the Monte Carlo results[9] shown in Fig. 3, in the points to the right of $6/g^2 = 6.0$. (The point at $6/g^2 = 6.6$ is believed to suffer from systematic errors and should be ignored.)

Now we can eliminate Λ_L and quote a value for the ratio $T_{\mathrm{tr}}/\sqrt{\sigma}$. This is an improvement over $T_{\mathrm{tr}}/\Lambda_{\overline{MS}}$ because σ is known quite well from quarkonium spectra and from the

FIGURE 2
Ratio of the transition temperature $T_{\rm tr}$ to the lattice scale parameter Λ_L in the pure SU(3) gauge theory. Monte Carlo data of Gottlieb *et al.*[5], with $T_{\rm tr}$ calculated via two-loop scaling. The line is the result of extrapolating to a constant.

FIGURE 3
Ratio of the string tension σ to the square of the lattice scale parameter Λ_L in the pure SU(3) gauge theory. Monte Carlo data of Barkai, Moriarty, and Rebbi[9], with σ calculated via two-loop scaling. The line is the result of extrapolating to a constant.

Regge slope [$\sigma \sim (400~\mathrm{MeV})^2$], and one can argue that it should not change much as one adds quarks to the pure gauge theory. The result, taking account of the conflicting Monte Carlo data, is

$$T_{\mathrm{tr}} = (200 - 230)\ \mathrm{MeV}. \tag{9}$$

A historical note: When $T_{\mathrm{tr}}/\Lambda_L$ was thought to have reached a continuum limit in the region $6/g^2 < 5.7$, the ratio σ/Λ_L^2 appeared to have settled down as well[10]. This was not fortuitous: it was due to the fact that the theory's β function[11] passed through its perturbative value before dipping below it in the region $5.7 < 6/g^2 < 6.0$. What was fortuitous was that the ratio T_{tr}/σ took the same value then as is quoted now, yielding the same result (9). Thus the bloody battles fought over the last two years have resulted in no ground gained by either side, but with high hopes for a lasting peace at the current cease-fire lines. A note of caution, however, is being sounded by Philippe de Forcrand, who tells me he has results for σ/Λ_L^2 which show a new drift for values of $6/g^2$ to the right of the points shown in Fig. 3. Oh, well. See his talk for details. For yet more recent results on $T_{\mathrm{tr}}/\Lambda_L$, see Tony Kennedy's talk.

It is important to note that lattice calculations this deep in the scaling regime require supercomputer power, even though we are still talking about the pure gauge theory. The early days, when VAX-scale calculations were interesting, are no more.

4. SIMULATIONS OF FULL QCD

When quarks are included in the dynamics of the lattice gauge theory, even rough first calculations cannot be attempted without supercomputers. Two years ago, reliable results were thought to lie in the dim future. This is still the case if one takes a strict definition of the word "reliable"; nevertheless, I can report exciting progress in simulating full QCD. As will be clear, we have entered a period in the study of QCD which is roughly comparable to the early days of the study of pure gauge theories.

4.1. Scaling and the continuum limit

The simplest fermionic operator to evaluate in QCD is the scalar bilinear $\bar{\psi}\psi$. This quantity is non-zero (at $T = 0$) in the limit of zero quark mass because of the spontaneous breaking of chiral symmetry: the origin of the nearly massless pion, soft pion theorems, and the rest of PCAC. Equation (3) takes the form

$$\langle \bar{\psi}\psi \rangle = a^{-3} f(g), \tag{10}$$

and a finite limit as $a \to 0$ requires* that $f(g) \propto a(g)^3$. In Fig. 4 I show Monte Carlo data obtained at great expense by Kogut[12,13]. The theory is QCD with 4 flavors of quarks, and the points are extrapolations to the $m = 0$ limit. The last three points lie, within error bars, on a scaling curve, which is a straight line if one ignores the power-law factor in (5).

* Not quite. $\bar{\psi}\psi$ actually does *not* have a finite continuum limit, but rather scales with an anomalous dimension. This is not discernible in the Monte Carlo results because of large error bars.

These are the *first* results showing scaling behavior in full QCD; they are the product of large amounts of supercomputer time expended on running the "hybrid microcanonical" algorithm[14].

FIGURE 4
Logarithm of the chiral order parameter, extrapolated to zero quark mass, in 4-flavor QCD with Susskind fermions; Monte Carlo data of Kogut[13]. The line is the behavior expected from one-loop scaling in the continuum limit for a theory with four flavors, ignoring anomalous dimensions.

4.2. High temperature

In Fig. 5 I show Kogut's[13] data for $\langle\bar{\psi}\psi\rangle$ in finite-temperature QCD. Again, this is the theory with 4 degenerate light flavors, but the points shown this time are actual Monte Carlo data for a small non-zero quark mass. Recall that varying the coupling g means varying the effective lattice spacing a via (4), and hence gives a way of adjusting the temperature via $T = (N_t a)^{-1}$. It is easy to verify that increasing $6/g^2$ means increasing T.

The region in Fig. 5 marked by the double arrow around $6/g^2 = 5.17$ is the finite-temperature "phase transition." To the left of this region, $\langle\bar{\psi}\psi\rangle$ takes large values not very

FIGURE 5

The chiral order parameter $\langle\bar{\psi}\psi\rangle$ and the Wilson line WL (the confinement order parameter) in 4-flavor QCD with light Susskind fermions at $T \neq 0$; Monte Carlo data of Kogut[13].

different from its value at $T = 0$. To the right, $\langle\bar{\psi}\psi\rangle$ is small, and would probably be zero if the quark mass were zero: this is chiral symmetry restoration, which is expected to be diagnostic of the high-temperature phase in QCD with light quarks. Also plotted in Fig. 5 is the Wilson line (denoted WL) which tests confinement physics. It rises steeply in the vicinity of the "transition," indicating dramatic variation in the $q\bar{q}$ potential.† Assuming scaling according to (7) gives

$$T_{\text{"tr"}}/\Lambda_{\overline{MS}} = (2.14 \pm .10) \tag{11}$$

for QCD with four flavors.

The reason I have put "phase transition" in quotes is that the data in Fig. 5 do not show one! There is no sign, at any sufficiently small (but non-zero) quark mass, of critical fluctuations or phase metastability as T is raised through the relevant regime; at most, there may be a very weak first-order transition hiding in the apparently smooth variation

† Unlike the situation in the pure gauge theory, $\langle WL\rangle$ is non-zero even in the low temperature phase: there is no absolute quark confinement even for low T because of $q\bar{q}$ pair production.

of $\langle\bar{\psi}\psi\rangle$ and $\langle WL\rangle$. Kogut's data, derived from the hybrid microcanonical algorithm, agree in detail with results obtained by Gavai[15] using a pseudofermion Metropolis algorithm (on a somewhat smaller lattice); see Rajiv Gavai's talk for a summary.

The conclusion that there is *no* phase transition between high- and low-temperature regimes contradicts the earlier results of Fucito *et al.*[16] (also from the pseudofermion Metropolis method), who claimed to have encountered a strong first-order transition for all small and intermediate values of the quark mass.* Gavai and Karsch[17] have given detailed criticism of the earlier work, the gist of which is that Fucito *et al.* didn't simulate for long enough to let the lattice come to equilibrium. Thus the hysteresis did not originate in a true first-order transition.

On the other hand, Fukugita and Ukawa[18], calculating with a Langevin algorithm, also claim to have seen a first-order transition for sufficiently light quark masses. One may argue with the interpretation of their data; indeed, here also one may argue that the simulations never reached equilibrium. In any case, reconciliation of their results with those of the other groups will take some time.

To be fair, those who see first-order transitions may reply to their detractors that the other simulations were *too* long, and that any theory on a lattice of finite size will show rounding of a phase transition, given enough time. There are always ambiguities in analyzing Monte Carlo results from finite lattices. Nevertheless, the burden of proof seems at present to be upon the advocates of the transition.

The current overall picture of finite-temperature QCD is summed up in Fig. 6. At the top of the diagram, at $m_q = \infty$, is the pure gauge theory, where there is a first-order confinement transition. Work done with the hopping parameter expansion[19], which is an expansion about the large-m_q limit, shows that the transition persists into the phase diagram, and that eventually a critical point is reached beyond which the high- and low-temperature phases are continuously connected. The simplest interpretation of Kogut's data is that there is a critical point (i.e., a continuous transition) at $m_q = 0$, with no singularities at all encountered as the temperature is varied at small but finite quark mass.

Unfortunately, there is a real problem with this picture. General arguments[20] lead to the prediction that there should be a *first* order transition for $m_q = 0$ when the number of flavors is three or greater. Three possible resolutions come to mind. One is that there is actually a weak first order transition hiding in the finite resolution of Kogut's Monte Carlo data. Another possibility is that there is a first order transition at $m_q = 0$ but that it vanishes quickly upon moving to $m_q \neq 0$, so that it cannot be seen in the finite-mass Monte Carlo simulations. In either case, the likely *true* phase diagram of QCD would have a first-order transition at $m_q = 0$ which continues into the diagram and ends in a critical point, much like the behavior of the $m_q = \infty$ transition.

The third possibility is somewhat more sinister. It rests upon the observation that all

* This work dealt with QCD with *three* flavors, reduced from four via an approximation valid in perturbation theory.

FIGURE 6

Summary of our knowledge of the phase diagram of QCD in the temperature–quark-mass plane. The line is a first-order transition, and the crosses are critical points.

simulations done to date at $T \neq 0$ have defined the lattice quark fields with the Susskind prescription. Within this scheme, the statement that there are four flavors has meaning only close to the continuum limit. The lattice theory is not symmetric under the chiral SU(4) $\times$ SU(4) symmetry group of the continuum theory, but only under an Abelian subgroup. It is quite possible that the theory with the Abelian symmetry undergoes a continuous transition, which will become first order as the continuum limit is approached and the full symmetry restored. Such worries can be laid to rest either by working closer to the continuum limit (that is, on larger lattices), or by doing parallel studies with Wilson fermions. See Akira Ukawa's talk for an example of the latter.

For further discussion of Kogut's results, I refer you of course to John Kogut's talk this afternoon.

4.3. The problem of finite chemical potential

Simulating QCD with non-zero chemical potential coupled to the baryon number is obviously very desirable for the study of quark matter. The major formal problem standing in the way of such work has nothing directly to do with the lattice cutoff, and is easy to explain. Starting from the path integral expression (1) for the partition function of QCD, we may integrate over the quark field variables ψ and $\bar{\psi}$ to get

$$Z = \int [dA_\mu] \exp\left(-\frac{1}{4}\int d^4x\, F^a_{\mu\nu} F^a_{\mu\nu}\right) \mathrm{Det}(\not{D} + m + \mu\gamma_0)^{n_f} \ . \tag{12}$$

The argument of the determinant is the Dirac operator (including the gauge coupling and the chemical potential μ), and n_f is the number of flavors. Since only the gauge field A_μ is left in (12), one may define an effective action, writing

$$Z = \int [dA_\mu] \exp\left[-S_{\text{eff}}(A_\mu)\right] \ . \tag{13}$$

So far everything is well defined. In particular, the fact that the determinant in (12) may make S_{eff} complex is no cause for worry.

The catch comes when one contemplates doing Monte Carlo simulations. The steps leading to (12) and (13) are in fact the first steps in all numerical calculations: One always integrates out the fermions analytically and leaves the gauge field for numerical work. All Monte Carlo methods depend on being able to treat $\exp[-S_{\text{eff}}(A_\mu)]$ as a (positive-definite) probability measure for the configuration of A_μ. Thus these methods cannot even be formulated when $\exp(-S_{\text{eff}})$ is complex. Unfortunately, this is exactly the case when $\mu \neq 0$.

Two approaches to this problem have been taken recently. The first is the most straightforward: *Throw away* the imaginary part of S_{eff}. I show in Fig. 7 the rough results of Berg *et al.*[21] They worked with a hopping parameter expansion of the fermion determinant, which is valid for large quark masses, and calculated such quantities as the Wilson line, $\langle\bar{\psi}\psi\rangle$, and the thermodynamic energy density ϵ as functions of T and μ. The points shown in the figure mark regions of rapid variation in these quantities, and do *not* necessarily indicate a phase transition. The neglect of $\operatorname{Im} S_{\text{eff}}$, the hopping parameter expansion, and the method of locating the "phase boundary" are all reasons to feel that these results are no more than qualitatively descriptive of the situation in real QCD. J. Engels will discuss this work further in his talk.

FIGURE 7
Results of Berg *et al.*[21] for the boundary between confining and plasma regimes in the temperature–chemical potential plane.

The second approach, which may gain more popularity in the future, is to use a Langevin process to simulate the partition function in place of standard Monte Carlo methods. The Langevin method allows continuing hitherto real parameters to complex values[22]. Theoretical justification of this continuation is still lacking (at least, outside of perturbation theory), but that is of course not a fatal shortcoming. The method has been applied[23] to a model resembling QCD with $\mu \neq 0$, but not yet to QCD itself. Presumably nobody wants to invest considerable amounts of computer time in a questionable algorithm.

For further news (and further problems[24]) regarding finite chemical potential studies, I refer you to Frithjof Karsch's talk this afternoon.

5. PHYSICS OF HIGH-TEMPERATURE QCD

Now I shall leave the glamorous world of incredibly expensive simulations of full QCD, and discuss more mundane questions of physics. These questions are posed well in the pure gauge theory, and answers may be pursued by the methods of lattice gauge theory. It is likely that both the questions and the methods to address them will pass on to the full theory when adequate computer power is available.

5.1. What *is* the quark-gluon plasma?

The fact that high-temperature QCD is far more complicated than high-temperature QED has been appreciated for quite some time[25]. I would begin by posing the following riddle (suggested by Janos Polonyi):

Consider the energy density of a black body:

$$\epsilon = \gamma \frac{\pi^2}{15} T^4, \tag{14}$$

where $\gamma = 1$ for photons and $\gamma = 8$ for free gluons. An old argument states that since QCD is asymptotically free, the coupling goes to zero logarithmically with increasing temperature, and thus gluons should indeed be free in the high-temperature limit. Indeed, Monte Carlo results for the pure gauge theory[26] have confirmed the validity of (14) for high-temperature glue.

Let us reexamine the old argument, however. An effective theory may be derived[27] for gluons in the high-temperature limit. This theory is defined in *three* Euclidean dimensions, and the action takes the form

$$S = \frac{1}{4} \int d^3x \, \frac{1}{g^2 T} F^a_{ij} F^a_{ij} + (A_0 \text{ terms}), \tag{15}$$

where I have picked out the self-interaction of the spacelike glue fields. (Recall that in a non-Abelian theory the field strength F^a_{ij} is non-linear in the gauge potential A^a_μ.) This self-interaction is precisely that of three-dimensional QCD, which is believed to *confine* color: There should be no free gluons! To be precise, the spectrum of states that contribute to the thermodynamic partition function should bear no resemblance to a set of eight massless, propagating, vector particles. What then is the meaning of the Monte Carlo results?

The resolution[28] of this particular paradox is fairly straightforward (once Nathan Weiss explained it to me). High-temperature QCD is a system with several scales. ϵ is dominated by physics at an energy scale on the order of T. Electric screening (Debye screening) occurs at the Debye scale, given perturbatively by gT. Three-dimensional confinement physics sets in at the scale g^2T, which for high temperatures (small effective coupling) is much smaller than T. Thus a different number of degrees of freedom is seen at different scales, and the high-momentum modes which dominate the energy density should be unaffected by the three-dimensional non-Abelian dynamics.

If ϵ is dominated by high-momentum degrees of freedom, can it be calculated order by order in perturbation theory? The answer[25] is a resounding *no*. The non-Abelian dynamics sneak into perturbative calculations and introduce serious infrared divergences, even in the large T limit. Furthermore, perturbation theory gives no guidance regarding separation of physical scales as $T \rightarrow T_{\rm tr}$, since then the effective coupling $g(T) \sim 1$, implying $gT \sim g^2T \sim T$.

The same problems[29] appear in the Debye screening length λ_D. In QED, the one-loop result $\lambda_D \sim (eT)^{-1}$ is not seriously affected by higher-order terms; in particular, there is no mixing of the screened electric field with the unscreened magnetic field. This is almost certainly *not* the case in QCD. I show in Fig. 8 Monte Carlo data[30] for the screening mass $\mu_D \equiv 1/\lambda_D$ in the pure gauge theory. The data are not really good enough to differentiate among $\mu_D \sim T$, $\mu_D \sim gT$, or $\mu_D \sim g^2T$; the fitted curve is $\mu_D \propto gT$, but with a coefficient some 30% smaller than the one-loop perturbation theory result. It is apparent that the high T limit where perturbative calculations are relevant is approached slowly, if at all. Earlier work[31] led to a similar conclusion at high T, but showed markedly different behavior as $T \rightarrow T_{\rm tr}$ from above; presumably the later results, obtained with much longer runs, are more reliable.

5.2. Thermodynamics and hydrodynamics

The calculation of thermodynamic properties of QCD was an early aim of lattice Monte Carlo simulations. I will show you recent results for the energy, the pressure, the entropy, and the speed of sound in hadronic and quark matter, and for ratios relevant to hydrodynamical models of high energy nucleus-nucleus collisions.

The energy density ϵ and the pressure P are of obvious physical interest; they can be written in terms of local operators whose measurement via Monte Carlo is straightforward. The entropy density s is related in phenomenological theories to the total particle multiplicity; it turns out that it can be calculated via numerical integration of Monte Carlo data. Finally, the speed of sound must be calculated from the formula

$$v_s^2 = \frac{dP}{d\epsilon} \qquad (16)$$

which requires numerical *differentiation*, a grisly prospect.

The results I show in Figs. 9–12 are from the work of Redlich and Satz[32] who reanalyzed existing Monte Carlo output[33]. The effects of dynamical quarks were included qualitatively

FIGURE 8
The electric screening mass in the pure SU(3) gauge theory, calculated by DeGrand and DeTar[30]. The curve is $M \propto g(T)T$.

via a low-order hopping parameter expansion. I remind you that this is an expansion about the $m_q = \infty$ limit; Redlich and Satz decline to supply statistical error bars for their results because the systematic errors in the expansion are likely to be severe.

P, ϵ, and s are shown in Fig. 9. The steep rise is the would-be phase transition, where color degrees of freedom are liberated. (There is no phase transition for sufficiently light quarks in the hopping parameter expansion[19].) I believe the statistical errors are not large.

Perhaps more interesting is the speed of sound v_s, shown in Fig. 10. If there is a first order transition, v_s should be zero there but non-zero, discontinuously, on either side. At a continuous transition one expects a smooth drop to zero from either side, and if there is no phase transition at all, there ought to be a residual dip. This is indeed seen in the Monte Carlo results, but the error bars from numerical differentiation are probably large: In the earlier work of Gavai and Gocksch[34] on the pure gauge theory, the error bars were almost as large as the dip.

Finally, I show in Figs. 11 and 12 two quantities used in hydrodynamical models. The ratio $P/(P + \epsilon)$ has been related to the collective transverse expansion of the central rapidity region[35], and it is seen to take a dip in the transition region. This is a tantalizing hint of slowed expansion, and hence longer lifetime, in collisions where plasma has been

FIGURE 9

Pressure P, energy density ϵ, and entropy density s of hot QCD, calculated via Monte Carlo in the hopping parameter (large quark mass) expansion[32].

produced. The ratio ϵ/s is proportional[36] to the average transverse momentum p_T of emerging secondaries, and is in turn a measure of temperature, since $\epsilon \sim T^4$ and $s \sim T^3$ in ideal gases. Note the change in slope in the transition region.

The approximations used in this work are rather crude, and the statistics are not sufficient; it is indicative of the possibilities for future applications of the more sophisticated Monte Carlo algorithms.

6. SUMMARY AND PROSPECTS

Concerning the status of lattice gauge theory, I have brought you both good news and bad news.

- The phase transition in the pure gauge theory seems to be under control. That's the good news. The bad news is that it took 6 years and the CYBER 205 to beat it into shape. Amid, and indeed before, the euphoria surrounding the early Monte Carlo results, Wilson[37] warned that lattice gauge theory would not yield believable results until a difficult renormalization group program had come to fruition. Events seem to have borne him out[38].
- Full QCD is giving numbers sooner than was expected two years ago. The Monte Carlo data for full QCD are about as good as the earliest data for the pure gauge theory.

FIGURE 10
The speed of sound in hot QCD

That's good news. But whereas Monte Carlo methods for the pure gauge theory allowed anybody with a CDC 6600, or even a VAX, to contribute to further progress, for full QCD the simplest calculations require supercomputer power, and plenty of it. It will take a while before any numbers settle down to hard predictions. One can hope for (and indeed, expect) dramatic advances in supercomputer technology and in algorithms, but for now, QCD is a rich man's game.

- High temperature QCD is still a mess. Sorry, here I started with the bad news. But there *is* no good news yet. Neither perturbation theory nor Monte Carlo work has yet given a reliable picture of the excitations of the high-temperature phase. The problem is, indeed, becoming urgent: What kind of quark-gluon plasma are we to look for at RHIC?

I will end on an optimistic note. While current lattice results for various phenomenologically interesting quantities are extremely rough, they do open the door towards more complete calculations. There *is* good work to be done in the near future. We can look forward to applications of state-of-the-art QCD programs to questions of interest for RHIC. Even if the first results will not be the last word, we will begin to define the parameters of the quark-gluon plasma.

FIGURE 11
The ratio $P/(P+\epsilon)$, from the data in Fig. 9

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FIGURE 12
The ratio of energy to entropy in hot QCD, from the data in Fig. 9.

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