An attractive nucleon-nucleon spin-orbit force from skyrmions with dilatons

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Abstract:- Within the skyrmion approach for the nucleon-nucleon force, difficulties have been experienced in obtaining an isoscalar attractive spin-orbit potential, in parallel to the problems of finding attraction in the isoscalar central potential. We here study the spin-orbit force using a skyrmion with four- and six-derivative stabilizing terms in the lagrangian as well as with the crucial addition of a dilaton. With these features present the spin-orbit force proves to be attractive as does the central potential. In the absence of the dilaton, attraction can also be found for the spin-orbit potential but only at the expense of a greatly over-emphasized term with six derivatives and a continuing absence of attraction in the central potential.

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In the mid-1980s there was a great revival of interest in Skyrme's original idea [1] whereby baryon physics is approximated by topological solitons. Very soon thereafter a number of attacks were made on the two-nucleon problem using skyrmions. The general skyrmion approach has been reviewed extensively [2–13] and particular emphasis on studies of the NN system is given in refs. [3,4,8–13]. The key difficulty in describing the NN force using skyrmions proves to be the subtle issue of finding attraction in the isoscalar central potential. This does not easily emerge [3,4,8–12] from the simplest approaches based on the product ansatz in which it is assumed that the solution for baryon number $B = 2$ is well approximated by the product of two solutions for $B = 1$. Eventually several possible mechanisms yielding central attraction were found. These included the use of a full numerical solution for the $B = 2$ system [12,13] or alternatively the admixture of higher resonances in the projected state for each of the two baryons [9,10]. These two approaches may not be that dissimilar since both attempt to include significant distortion of one nucleon due to the presence of the other as suggested also in calculations based on the nonrelativistic quark model [14]. In the two cases, of course, quite different degrees of freedom are chosen to express this situation.

A different mechanism through which attraction is obtained has to do with the coupling of a dilaton to the skyrmion. It has long been known [15] that an effective chiral lagrangian can be made to mimic the scale breaking of quantum chromodynamics by introducing a dilaton field tailored to yield the same trace anomaly as possessed by QCD. It is plausible that the introduction of such an additional scale will help towards the solution of the problem of NN central attraction since it serves to sharpen the surface of the baryon described by the skyrmion. This in turn prevents the repulsive nature of the $B = 2$ system for small interbaryon separations from overwhelming the possibility of attraction in the range of 2 fm or so. Thus there is appreciable central attraction for skyrmions deriving from a lagrangian that is coupled to a dilaton [16,17].

More recently considerable effort has been expended in studying the NN spin-orbit force with skyrmions [18–23], in particular the isoscalar part thereof. Once again it has not proved possible to obtain the correct, negative sign for this component with the simple product ansatz [19,20]. This led Riska and Schwesinger [21] to suggest that a term with six derivatives often used to augment skyrmion stabilization [24–26] may bring about an attractive isoscalar spin-orbit force, although an unusually large coupling constant for this term was required to accomplish this [23]. They also suggested that the inclusion of a dilaton field would increase this attraction. The present study is a detailed working out of that idea.

Our point of departure is the skyrmion lagrangian with four- and six-derivative terms and dilaton coupling to produce scale invariance,

$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}_{2\,\mathrm{dilaton}} + \mathcal{L}_{2\,\mathrm{skyrmion}} + \mathcal{L}_4 + \mathcal{L}_6 - V(\phi) \\
&= \mathrm{e}^{2\phi}\left[\frac{1}{2}\Gamma_0^2 \partial_\mu \phi \partial^\mu \phi - \frac{F_\pi^2}{16}\mathrm{tr}(L_\mu L^\mu)\right] + \frac{1}{32e^2}\mathrm{tr}[L_\mu,\, L_\nu]^2 \\
&- \epsilon_6^2 \mathrm{e}^{-2\phi} B_\mu B^\mu - \frac{C_G}{4}[1 + \mathrm{e}^{4\phi}(4\phi - 1)].
\end{aligned}
\tag{1}
$$

Here $L_\mu \equiv U^\dagger \partial_\mu U$ and $B^\mu \equiv -(\epsilon^{\mu\alpha\beta\gamma}/24\pi^2)\mathrm{tr}(L_\alpha L_\beta L_\gamma)$, where $U(\vec{r},t)$ is the unitary SU(2) chiral field, F_π is the pion decay constant (with experimental value 186 MeV), e is the Skyrme parameter, and ϵ_6 is the coefficient of the six-derivative repulsive term. In ref. [25] identification is made between $\mathcal{L}_6$ and ωNN coupling (through a term closely related to $\mathcal{L}_6$ but containing a single trace over six factors of L_μ rather than the two traces over three L_μs shown here) and it is determined that $\epsilon_6 = g_\omega^2/2m_\omega^2$, where the ωNN coupling constant is empirically [25] $g_\omega^2/4\pi \sim 10$, the value we have taken here, and the ω mass is $m_\omega = 782$ MeV. The dilaton parameters [15] are chosen as $\Gamma_0 = 137$ MeV and $C_G = (121\ \mathrm{MeV})^4$.

The static $B = 1$ solution for eq. (1) is taken in the usual hedgehog form $U_0(\vec{r}) = \exp[i\vec{\tau}\cdot\hat{\vec{r}}F(r)]$ and for the $B = 2$ system we use the product ansatz

$$
U_{B=2} = A_1(t)U_0(\vec{r} - \vec{r}_1(t))A_1^\dagger(t)\, A_2(t)U_0(\vec{r} - \vec{r}_2(t))A_2^\dagger(t), \tag{2}
$$

where $\vec{r}_1(t) = \vec{r} + \vec{R}(t)/2$ and $\vec{r}_2(t) = \vec{r} - \vec{R}(t)/2$ so that $\vec{r}_1(t) - r_2(t) \equiv \vec{R}(t)$ is the dynamical separation between the two nucleons, and $A_1(t)$ and $A_2(t)$ are the collective rotations of skyrmion 1 and 2 from which projection onto nucleon states is generated. The dilaton for $B = 2$ is taken to be additive as usual [16],

$$
\phi_{B=2} = \phi_1 + \phi_2. \tag{3}
$$

Both the assumptions of a product ansatz for the skyrmion and of an additive form for the dilaton are expected to break down for small separation distances but this is not of overly great concern here since we are interested in the range $R > 1$ fm for the central and spin-orbit potentials. The evaluation of NN potentials proceeds as usual by calculating the energy of the $B = 2$ system from the insertion of eq. (2) into eq. (1), projecting onto nucleon states, and identifying the relevant part of the energy (e.g., in the cases shown here only isoscalar components are considered). For the central potential V_C it is of course necessary to subtract the energy $V_\mathrm{C}(R \to \infty)$.

Forms for the NN isoscalar spin-orbit potentials from skyrmions given in the literature have been contradictory, nor do we get complete agreement with the reported results. We therefore attempt to sketch here a few key steps in the calculation. We use the convenient form [19]

$$\dot{U}(\vec{r}-\vec{r}_k(t)) = A_k(t)\left(\frac{i}{2\lambda}\,[\vec{\tau}\cdot\vec{j}_k,\, U_0(\vec{r}-\vec{r}_k)] - \frac{\vec{p}_k}{M}\cdot\nabla U_0(\vec{r}-\vec{r}_k)\right)A_k^\dagger(t), \qquad k = 1, 2, \tag{4}$$

where M and λ are the skyrmion mass and moment of inertia [2–13] trivially modified by the dilaton such that parts arising from $\mathcal{L}_{2\,\mathrm{skyrmion}}$ acquire a factor $\mathrm{e}^{2\phi}$ and those coming from $\mathcal{L}_6$ have $\mathrm{e}^{-2\phi}$. In eq. (4), $\vec{j}_k$ is the angular momentum operator for nucleon k (i.e., $\vec{j}_k \to \frac{1}{2}\vec{\sigma}_k$, where $\vec{\sigma}_k$ is the set of Pauli matrices for nucleon k). We also are aided by the projection theorem [19]

$$\langle N'|A_k\vec{\tau}A_k^\dagger|N\rangle = -\frac{1}{3}\langle N'|\vec{\sigma}_k(\vec{\tau}\cdot\vec{\tau}_k)|N\rangle, \qquad k = 1, 2, \tag{5}$$

in which the subscripted operators refer to the nucleon space while $\vec{\tau}$ is the skyrmion SU(2) matrix. Reference [19] also gives explicitly the expansion of $\mathcal{L}_2$ and $\mathcal{L}_4$ in terms of $\dot{U}_k$ and ∇U_k. For the *isoscalar* spin-orbit case the overall traces over the combined space of the $B = 2$ skyrmion can be broken down into separate traces for the $k = 1$ and $k = 2$ factors. It is important to note that in the presence of the dilaton the final result contains terms referring to a single nucleon of the form $\vec{\sigma}_1 \times \vec{r}_1 \cdot \vec{p}_1 \to \frac{1}{2}\vec{\sigma}_1 \cdot \vec{L}(1 + 2z/R)$, after averaging over the azimuthal angle taken with respect to the direction of $\vec{R}$, and "crossed" terms referring to both nucleons of the form $\vec{\sigma}_1 \times \vec{r}_1 \cdot \vec{p}_2 \to -\frac{1}{2}\vec{\sigma}_1 \cdot \vec{L}(1 + 2z/R)$, where $\vec{L} \equiv \vec{R} \times \vec{P}$ with $\vec{P} \equiv (\vec{p}_1 - \vec{p}_2)/2$, and z is the component of the skyrmion variable along the direction of $\vec{R}$. One also encounters from $\mathcal{L}_4$ combinations like $\vec{P}\cdot(\vec{r}+\vec{R}/2)(\vec{r}+\vec{R}/2)\cdot(\vec{\sigma}_2 \times (\vec{r}-\vec{R}/2)) \to -\frac{1}{2}(r^2 - z^2)(\vec{\sigma}_2 \cdot \vec{L})$, where again the arrow indicates averaging over the azimuthal angle.

With the above prescriptions the calculation of the potentials reduces to straightforward, if somewhat tedious, calculations of traces. Our result for the isoscalar spin-orbit potential is then

$$V_{\mathrm{SO},I=0} = \frac{\vec{S}\cdot\vec{L}}{\lambda M}(V_2 + V_4 + V_6), \tag{6}$$

where $\vec{S} \equiv \frac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2)$ and, in parallel to the skyrmion components of the lagrangian in eq. (1),

$$V_2 = \frac{F_\pi^2}{8}\int \left(\mathrm{e}^{2(\phi_1+\phi_2)} - \mathrm{e}^{2\phi_1}\right)\frac{s_1^2}{r_1^2}(1 + 2z/R)d\vec{r}, \tag{7a}$$

with $s_k \equiv \sin F_k,\ k = 1,2$;

$$V_4 = \frac{1}{2e^2}\int \left[\frac{s_1^2}{r_1^2}\left(F'^2_2 + \frac{3s_2^2}{r_2^2}\right)(1 + 2z/R) + \frac{s_2^2}{r_1^2 r_2^2}\left(F'^2_1 - \frac{s_1^2}{r_1^2}\right)(r^2 - z^2)\right]d\vec{r}, \tag{7b}$$

and

$$V_6 = -\epsilon_6^2\left[\int \mathrm{e}^{-2(\phi_1+\phi_2)} B_1^0 B_2^0 d\vec{r} - \int \left(\mathrm{e}^{-2(\phi_1+\phi_2)} - \mathrm{e}^{-2\phi_1}\right)(B_1^0)^2(1 + 2z/R), \tag{7c}$$

where $B_k^0 \equiv -F_k' \sin^2 F_k / 2\pi^2 r_k^2,\ k = 1,2$ is the baryon density for nucleon 1 or 2.

The calculational procedure now consists in solving the equations of motion derived from $\mathcal{L}$ of eq. (1) for hedgehog skyrmion and dilaton profiles. Parameters for the skyrmion are restricted by requiring reasonable values for the nucleon and Δ masses. These profiles are then inserted into eqs. (7) to produce the spin-orbit potential (or into the well-known expressions for the central potential—see refs. [3,9]). Results are shown in figs. 1 through 5. The first two of these show attraction of the correct magnitude as given by potentials fitted to the NN scattering data both for the spin-orbit and the central potentials. Both cases contain the six-derivative term $\mathcal{L}_6$ and dilaton coupling. Without the dilaton present (figs. 3 and 4) no attraction is found for V_{C}, as is expected from previous work on the NN system with skyrmions [3,9]. Figure 4 does show $V_{\mathrm{SO}} < 0$ as found in the work of ref. [21]. As there, this is achieved by exaggerating the importance of the six-derivative term in the lagrangian $\mathcal{L}_6$, in our case by ignoring $\mathcal{L}_4$ altogether. This seems to us to be artificial in that one expects [27] from $\pi\pi$ scattering that $e \approx 5 \pm 2$; it also leaves unsolved the sources of central attraction. Figure 5 uses $\mathcal{L}_4$, with a smaller value of e, and $\mathcal{L}_6$ and the dilaton; it fails to show attraction for V_{SO}. In fact for values of e less than about 9 we do not achieve $V_{\mathrm{SO}}(R) < 0$ in the range of NN separations of relevance here, $1 < R < 2$ fm. Although $e \geq 9$ is somewhat outside the range suggested by $\pi\pi$ scattering, the combination of the

$\mathcal{L}_6$ term and dilaton coupling does yield a reasonable, semiquantitative picture for the main ingredients of the NN force as derived from skyrmions.

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Figure captions:-

Fig. 1. Spin-orbit potential and central potential (dashed line for $r \geq 2$ fm) for skyrmion parameters $F_\pi = 143$ MeV and $e = 20.0$, yielding a nucleon mass $M_N = 942$ MeV and a Δ mass $M_\Delta = 1174$ MeV. Here $\mathcal{L}_4$, $\mathcal{L}_6$, and the dilaton are all present and both $V_{\rm SO}$ and $V_{\rm C}$ are attractive.

Fig. 2. Spin-orbit and central potentials for $F_\pi = 140$ MeV and $e = 9.0$, yielding $M_N = 976$ MeV and $M_\Delta = 1169$ MeV. Curves are as defined in fig. 1.

Fig. 3. Spin-orbit and central potentials for $F_\pi = 112$ MeV and $e = 4.84$ without dilaton coupling. Here $M_N = 978$ MeV and $M_\Delta = 1106$ MeV. Curves are as defined in fig. 1. Neither $V_{\rm SO}$ nor $V_{\rm C}$ is attractive.

Fig. 4. Spin-orbit and central potentials for $F_\pi = 140$ MeV using only $\mathcal{L}_6$ to stabilize the skyrmion (i.e., $\mathcal{L}_4 \equiv 0$) and without dilaton coupling. The mass values that emerge are $M_N = 942$ MeV and $M_\Delta = 1251$ MeV. Curves are as defined in fig. 1.

Fig. 5. Spin-orbit and central potentials for $F_\pi = 112$ MeV and $e = 4.84$ with $\mathcal{L}_4$, $\mathcal{L}_6$, and the dilaton all present, yielding $M_N = 922$ MeV and $M_\Delta = 1048$ MeV. Curves are as defined in fig. 1.