Proton spin content from skyrmions

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Abstract:- It is well known that in lowest order the skyrmion model of the nucleon gives vanishing spin content. With new data indicating a proton spin content $\Delta\Sigma = 0.22 \pm 0.14$, it is an increasing challenge to find ways in which the skyrmion can move away from the null result. We show here that a particular term in the skyrmion lagrangian in SU(3) involving six derivatives of the field can, with plausible parameters, yield a spin content consistent with present experiment.

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Of the various models applied to the explanation of the original EMC measurement [1] of the spin content of the proton, it remains the distinction of the skyrmion that it implies [2] in its lowest-order statement a small—indeed, a vanishing—value for that quantity. (Reviews of the current general status of the spin-content problem are given in Refs. [3,4].) This, however, presented the Skyrme model [5] with a special challenge when the SMC experiment [6] recently found a spin content of $\Delta\Sigma = 0.22 \pm 0.14$. It has not proved easy, despite several attempts [7,8]—and notably the sustained effort of the group centered about Syracuse [9-11]—to move the Skyrme model away from its statement of vanishing, or very small, spin content. On the other hand the spin structure of the nucleon should allow a crucial test of the Skyrme model, which fits a large number of hadronic data successfully, but an important feature of which, namely, the specific linking of spin and isospin and its consequences for spin content, has not yet been fully assessed. If $\Delta\Sigma \approx 0$ is really an indisputable property of the Skyrme model in its general form, the spin data could rule it out definitively. The present paper shows that this is not the case and identifies a source of nonnegligible spin. This is found in the form of a term in an extended Skyrme lagrangian involving six derivatives of the field. In SU(3) the particular form we use is different from that considered in the past [12-14] for the stabilization of the skyrmion through an ω-like exchange, and, with reasonable parameters, leads to a spin content near that of experiment. This result may be viewed, in some sense, as a simpler, effective version of the Syracuse studies [9-11], which are based on a large number of couplings to various mesons.

The skyrmion is here described by the lagrangian

$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_{4,1} + \mathcal{L}_{4,2} + \mathcal{L}_{6,1} + \mathcal{L}_{6,2} + \mathcal{L}_{SB} \\
&= -\frac{F_\pi^2}{16}\mathrm{tr}(L_\mu L^\mu) + \frac{1}{32e^2}\mathrm{tr}[L_\mu, L_\nu]^2 \\
&+ \frac{\gamma_1}{4e^2}\mathrm{tr}(L_\mu L^\mu L_\nu L^\nu) + \frac{\gamma_2}{8e^2}\mathrm{tr}(L_\mu L^\mu)\mathrm{tr}(L_\nu L^\nu) \\
&- \epsilon_1 \frac{g_\omega^2}{m_\omega^2}\mathrm{tr}(B^\mu B_\mu) - \epsilon_2 \frac{g_\omega^2}{2m_\omega^2}\mathrm{tr}(B^\mu)\mathrm{tr}(B_\mu) \\
&+ \left[\frac{F_\pi^2}{32}(m_\pi^2 + m_\eta^2)\mathrm{tr}(U + U^\dagger - 2) + \frac{\sqrt{3}F_\pi^2}{24}(m_\pi^2 - m_K^2)\mathrm{tr}(\lambda_8(U + U^\dagger))\right],
\end{aligned}
\tag{1}
$$

apart from an anomalous contribution to the η' mass which is not relevant for our purposes here. In the following we shall systematically refer to the terms by their corresponding subscripted forms as given in the first line of eq. (1). Here

$L_\mu \equiv U^\dagger \partial_\mu U$ and

$$B^\mu \equiv -\frac{\epsilon^{\mu\alpha\beta\gamma}}{24\pi^2} L_\alpha L_\beta L_\gamma, \tag{2}$$

where $U(\vec{r}, t)$ is the U(3) chiral field, F_π is the pion decay constant (with experimental value 186 MeV), and e is the Skyrme parameter.

Let us motivate our choice for the lagrangian in some detail: To describe the nucleon spin structure correctly even only at the qualitative level one should include terms which correspond to ω-exchange because the three-vector part of the ω couples to spin. As B_μ relates to the baryon current one consequently has to include terms quadratic in it. We shall show that the term $\mathcal{L}_{6,1}$, which contains only a single trace over field variables, modifies the spin content of the skyrmion substantially. In contrast $\mathcal{L}_{6,2}$, which is a product of two such traces, does not affect the spin (in line with the suggestion [9-11] of the OZI rule). Still higher-order terms or other sixth-order terms might modify the results quantitatively but not qualitatively; we therefore disregard them for the time being. We shall show that $\mathcal{L}_{6,1}$ generates naturally a nonzero skyrmion spin and this suggests that any complete treatment of the skyrmion will get $\Delta\Sigma \neq 0$.

Since we take sixth-order terms into account we also add the two fourth-order terms $\mathcal{L}_{4,1}$ and $\mathcal{L}_{4,2}$ for completeness. The first of these corresponds to a term which Ryzak [7] claimed to give a $\Delta\Sigma$ substantially different from zero, a claim our results do not support. These terms are notorious as they contain quartic time derivatives and also work to destabilize the skyrmion. No consensus exists on their interpretation and treatment. We regard it therefore as supportive of the straightforward interpretation of the Skyrme model that these terms are unimportant for $\Delta\Sigma$ in our calculation.

The parameters γ_1 and γ_2 in eq. (1) can be related to $\pi\pi$ scattering, and the terms $\mathcal{L}_{4,1}$ and $\mathcal{L}_{4,2}$ have been used in the past to increase NN attraction in the skyrmion (see [15] and, e.g., the review in Ref. [14]). The parameters $\epsilon_1 g_\omega^2/m_\omega^2$ and $\epsilon_2 g_\omega^2/m_\omega^2$ are coefficients of six-derivative terms $\mathcal{L}_{6,1}$ and $\mathcal{L}_{6,2}$, respectively, both of which have been used in the past as possible ω-coupling repulsive terms for stabilizing the skyrmion [12,13]. They are equivalent [13] in SU(2), but not in SU(3). (Terms with four derivatives that are equivalent in SU(2) but different in SU(3) are, of course, well known [16].) (In the past, a contribution to spin content from a term that would correspond in this classification to an eight-derivative part has been considered [8] and found to give a small contribution, but, to the best of our knowledge, the six-derivative term has thus far been bypassed.) The forms of the coefficients of $\mathcal{L}_{6,1}$ and $\mathcal{L}_{6,2}$ are selected to allow easy comparison with the ωNN coupling constant. We keep $\epsilon_1 + \epsilon_2 = 1$, absorbing overall strength into g_ω^2, with m_ω, the mass of the ω, fixed at its experimental value, so that $\mathcal{L}_{6,1} + \mathcal{L}_{6,2}$ at the SU(2) level continues to give the usual ω-like coupling. The flavor symmetry-breaking term $\mathcal{L}_{SB}$ in eq. (1) is well known [17] to be important for work with the SU(3) skyrmion.

In order to generate the proton spin content from $\mathcal{L}$ of eq. (1), we introduce the U(3) matrix

$$U = \exp\left[\frac{2i}{F_\pi}\left(\eta' + \sum_{a=1,8} \lambda_a \phi_a\right)\right], \tag{3}$$

where ϕ_a is the pseudoscalar octet and η' is the ninth pseudoscalar meson. As is well known [7,8], there will be no contribution to the spin content from a U(1) axial current that is a complete four-derivative, since this vanishes in producing $\Delta\Sigma$ as an integral over all space. We construct the axial current of interest to us here out of a term in the lagrangian of the form [8]

$$\mathcal{L}' = (2/F_\pi)\partial_\mu \eta' J^\mu \tag{4}$$

generated by varying η'. In the evaluation of this J^μ, we use the customary approach of collective coordinates for the time dependence [18]

$$U(\vec{r}, t) = A(t) U_0(\vec{r}) A^\dagger(t), \tag{5}$$

with the hedgehog embedded in SU(3) as

$$U_0 = \exp[i\vec{\lambda} \cdot \vec{r} F(r)], \tag{6}$$

where $F(r)$ is the profile function. It is well known [8] that the static hedgehog contribution to spin content vanishes on grounds of grand-spin symmetry, so that a nonzero value is obtained only through the collective-coordinate rotation. In evaluating the nonstatic component L_0 with quantization of the collective-coordinate transformation we encounter

$$A\dot{A}^\dagger = \frac{i}{2\alpha^2}\vec{\lambda} \cdot \vec{R} + \frac{i}{2\beta^2} \sum_{a=4,7} \lambda_a R_a + \dot{q}_i C_{i8}(q) \lambda_8, \tag{7}$$

where q_i is an SU(3) variable and the last term does not contribute in our expressions. The quantities α^2 and β^2 relate in the usual way to the SU(3) moments of inertia [17], and R_a, $a = 1, 2, \ldots, 8$, are the "right" SU(3) generators.

Since the terms $\mathcal{L}_{4,2}$ and $\mathcal{L}_{6,2}$ have been treated extensively in the literature, and, more to the point, do not contribute to the flavor-singlet axial current, we shall not repeat the well known results for them here. For $\mathcal{L}_{4,1}$ and $\mathcal{L}_{6,1}$ the new, additional static contributions to the skyrmion mass are

$$M_{4,1} = -4\pi \frac{\gamma_1}{2e^2} \int_0^\infty r^2\, dr \left(F'^2 + 2\frac{\sin^2 F}{r^2} \right)^2, \tag{8}$$

and

$$M_{6,1} = 288\pi \left(\frac{1}{24\pi^2} \right)^2 \frac{\epsilon_1 g_\omega^2}{m_\omega^2} \int_0^\infty r^2\, dr \left(F' \frac{\sin^2 F}{r^2} \right)^2. \tag{9}$$

The corresponding new contributions to the total moment-of-inertia parameters α^2 and β^2 are

$$\alpha^2_{4,1} = -16\pi \frac{\gamma_1}{3e^2} \int_0^\infty r^2\, dr \sin^2 F \left(F'^2 + 2\frac{\sin^2 F}{r^2} \right), \tag{10}$$

$$\beta^2_{4,1} = -4\pi \frac{\gamma_1}{e^2} \int_0^\infty r^2\, dr \sin^2 \frac{F}{2} \left(F'^2 + 2\frac{\sin^2 F}{r^2} \right), \tag{11}$$

and

$$\alpha^2_{6,1} = 384\pi \left(\frac{1}{24\pi^2} \right)^2 \frac{\epsilon_1 g_\omega^2}{m_\omega^2} \int_0^\infty r^2\, dr \left(\frac{F' \sin^2 F}{r} \right)^2, \tag{12}$$

$$\beta^2_{6,1} = 64\pi \left(\frac{1}{24\pi^2} \right)^2 \frac{\epsilon_1 g_\omega^2}{m_\omega^2} \int_0^\infty r^2\, dr \sin^2 \frac{F}{2} \frac{\sin^2 F}{r^2} \left(2F'^2 + \frac{\sin^2 F}{r^2} \right), \tag{13}$$

The term $\mathcal{L}_{4,1}$ yields a contribution to the matrix element of the flavor-singlet axial current given by

$$\langle \mathrm{p} \uparrow | J^3_{4,1} | \mathrm{p} \uparrow \rangle = -2\pi \frac{\gamma_1}{3e^2} \int_0^\infty r^2\, dr \left[\frac{1}{4\alpha^4} F' \sin^2 F + \frac{1}{\beta^4} \left(F' - 2\frac{\sin F}{r} \right) \sin^2 \frac{F}{2} \right]; \tag{14}$$

this expression differs from that in [7] and leads to a smaller contribution to spin content for $\mathcal{L}_{4,1}$. The corresponding matrix element for $\mathcal{L}_{6,1}$ is

$$\begin{aligned} \langle \mathrm{p} \uparrow | J^3_{6,1} | \mathrm{p} \uparrow \rangle = & \frac{32\pi}{3} \left(\frac{1}{24\pi^2} \right)^2 \frac{\epsilon_1 g_\omega^2}{m_\omega^2} \frac{1}{\beta^4} \\ & \times \int_0^\infty r^2\, dr \sin^2 \frac{F}{2} \sin F \frac{1}{r} \left(F'^2 - F' \frac{\sin F}{r} + \frac{\sin^2 F}{r^2} \right). \end{aligned} \tag{15}$$

As has been pointed out [7], these calculations are "somewhat tedious but otherwise straightforward," and so to assure their reliability we have evaluated eqs. (10) through (15) both by hand and using a symbol-manipulation program. In arriving at these results, we have also used the feature of the "right" SU(3) algebra that $R_4^2+R_5^2-R_6^2-R_7^2 = R_3$, and $\langle \mathrm{p}\uparrow|R_3|\mathrm{p}\uparrow\rangle = -\frac{1}{2}$, as well as the identification of the spin content through $\Delta\Sigma = 2\langle \mathrm{p}\uparrow|J^3|\mathrm{p}\uparrow\rangle$. In $J^3_{6,1}$ only symmetric combinations of the R_as appear. On the other hand, $J^3_{4,1}$ contains both commutators and anticommutators. We drop the former on the basis of arguments [9] that in going from the classical to the quantal results for the rotation operators one must symmetrize them, whence the commutators do not contribute. This eliminates the term in the integrand of eq. (14) with the coefficient $1/\alpha^4$ and also divides the right-hand side of that equation by 2. Since the contribution of $J^3_{4,1}$ is in any event not very appreciable, our conclusions below are not changed significantly by this procedure.

While the choice of some of the parameters appearing in the lagrangian of eq. (1) is straightforward, others are not so easily fixed. The situation is eased somewhat by the realization that the contribution of the $\mathcal{L}_{4,1}$ term to the spin content is very small, on the order of 0.04 for a maximizing case with $\gamma_1 \sim 0.15$ and $e = 5$; efforts further to enhance that contribution led to a breakdown in the stability of the skyrmion solution. Thus, in the following, we drop $\mathcal{L}_{4,1}$ as well as $\mathcal{L}_{4,2}$, which also eliminates concern over terms in the lagrangian with higher than bilinear time derivatives. We then fix $g_\omega^2/4\pi = 10$, as implied by [13] ωNN coupling and by [12] the decay $\omega \to 3\pi$. Unfortunately, there seems to be no very direct way to determine the crucial coefficient of the $\mathcal{L}_{6,1}$ term, ϵ_1. The natural candidate would be to compare with $\eta' \to 5\pi$, but this vanishes dynamically for $\mathcal{L}_{6,1}$, and indeed experimentally [19] the branching ratio for this mode seems to be less than a percent or so. A transition that does not vanish dynamically is $\eta' \to \eta + 4\pi$, but this, of course, is kinematically forbidden as a decay. Thus we proceed by fixing F_π and e so as to produce reasonable values for nucleon properties and seeking a range for ϵ_1 that yields sizable spin content. With $F_\pi = 130$ MeV and $e = 20$, we have $M_N = 952$ MeV and $M_\Delta = 1,241$ MeV. The SU(2) result for the axial coupling constant is $g_A = 1.21$, in surprisingly good agreement with experiment given the usual experience with skyrmions [18]. The nucleon charge radius is 0.71 fm, and the magnetic moments are $\mu_{\mathrm{p}} = 2.04$ and $\mu_{\mathrm{n}} = -1.24$. We then find that $\epsilon_1 = -0.7$ yields a spin content of $\Delta\Sigma = 0.24$. The centroid of the masses of particles with nonzero strangeness then reaches roughly 1,650 MeV, after subtraction of the zero-point energy [17]. Compensation is possible by reducing F_π, but further adjustment of parameters does not seem worthwhile in the face of the poorly known value for ϵ_1. As a rough indicator of sensitivities, varying ϵ_1 between -0.6 and -0.8 causes $\Delta\Sigma$ to range between 0.18 and 0.32 and the strange-mass centroid to change between 1,610 MeV and 1,670 MeV. The reason for the rather large contribution to the spin content from the $\mathcal{L}_{6,1}$ term lies in the reduction of the total β^2 caused by the negative $\beta^2_{6,1}$. Thus the $\mathcal{L}_{6,1}$ term shows a possible source of spin content for the skyrmion well within the range of present experiment, but further precision in the theory requires a better determination of its coefficient.

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