THE NUCLEON IN THE NUCLEUS AS GIVEN BY SKYRMIONS

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After some ten years or more of effort towards understanding the nucleon-nucleon force in terms of skyrmions, the way may now be open to consider its application to various problems involving the manifestations of quantum chromodynamics in nuclear physics. We here consider two such studies: (i) aspects of color transparency as given by skyrmions and (ii) the treatment of nucleon swelling and shrinking in nuclei using skyrmions.

1 Introductory Remarks

1.1 Historical Perspective

During the last ten years or so, roughly ten groups around the world have pursued the issue of correctly describing the nucleon-nucleon force and systems with small numbers of nucleons using skyrmions.[a] In offering this modest contribution in honor of Professor Walter Greiner it was tempting to review the impressive progress that has recently been made in that direction. However, it is one of Professor Greiner's engaging qualities to be more oriented towards the future than the past, and at the opening of this conference he asked that the speakers not stint in addressing the future in their areas—a request which it is a pleasure to comply with here.

Thus we shall take as our point of departure the supposition that the skyrmion has become, or very soon will become, an adequate tool for qualitative and semiquantitative descriptions of few-baryon systems, so that the way is open to begin considering its use for insight into hadronic behavior in nuclei, a topic of increasing importance in a variety of studies of quantum chromodynamics (QCD) effects in nuclei. The first such topic to be considered here is

[a] This work has been extensively surveyed, practically every group involved having written a summary of the topic from its own perspective. We here note one review [1] which is close to the usages and approaches of this talk.

color transparency, reviewed, for example, in ref. [2], which involves very-high-momentum transfer processes within nuclei in a test of QCD dynamical issues. The second question relates to nucleon swelling and shrinking in nuclei, a matter that has received increasing attention over the last ten years or so [3,4,5,6,7]. More detailed discussions of both these topics are given in the review noted previously [1] and in the original papers [8,9].

2 Generalities on $B = 1$ Skyrmions

2.1 The Skyrme Lagrangian

It was pointed out a number of years ago by 't Hooft that in the limit of a large number of colors, $N_c \to \infty$, QCD reduces to a theory involving meson fields as the relevant degrees of freedom [10], while Witten pointed out that baryons can then be constructed out of these degrees of freedom as topological solitons [11]. This provided underpinnings within QCD for a very old and prescient idea of Skyrme [12] that baryons might be built up out of topological solutions of a lagrangian embodying chiral symmetry, namely,

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 = -\frac{F_\pi^2}{16}\mathrm{tr}(L_\mu L^\mu) + \frac{1}{32e^2}\mathrm{tr}[L_\mu, L_\nu]^2, \tag{1}$$

where F_π is the pion decay constant, with experimental value 186 MeV, and e is the Skyrme parameter which accompanies his stabilizing term with four derivatives. Here

$$L_\mu \equiv U^\dagger \partial_\mu U, \tag{2}$$

where $U(\mathbf{r}, t)$ is the chiral field, here taken in SU(2). For the $B = 1$ static problem we take a time-independent hedgehog solution,

$$U(\mathbf{r}, t) = \exp[i\boldsymbol{\tau} \cdot \hat{\mathbf{r}} F(r)], \tag{3}$$

where $F(r)$ is referred to as the profile function. Substituting eq. (3) into eq. (1) yields eventually an expression for the particle energy or mass in terms of the profile function,

$$\begin{aligned} M &= 4\pi \int_0^\infty r^2\, dr \left\{ \frac{F_\pi^2}{8}\left[\left(\frac{dF}{dr}\right)^2 + 2\frac{\sin^2 F}{r^2}\right] \right. \\ &+ \left. \frac{1}{2e^2}\frac{\sin^2 F}{r^2}\left[\frac{\sin^2 F}{r^2} + 2\left(\frac{dF}{dr}\right)^2\right]\right\}. \end{aligned} \tag{4}$$

From this one sees that in order for the mass to remain finite the profile function must approach $n\pi$ as $r \to 0$ and $m\pi$ as $r \to \infty$, where n and m are integers whose difference $n-m$ is a topologically conserved quantity that Skyrme identified as baryon number. Moreover the minimum reached by varying F in eq. (4) yields the Euler-Lagrange equation for the profile function,

$$\left(\frac{1}{4}\tilde{r}^2 + 2\sin^2 F\right)F'' + \frac{1}{2}\tilde{r}F' + \sin 2F F'^2 - \frac{1}{4}\sin 2F - \frac{\sin^2 F\,\sin 2F}{\tilde{r}^2} = 0, \tag{5}$$

in terms of dimensionless variables for which lengths are scaled by eF_π and energies by F_π/e. This dimensionless form will be particularly handy for discussing color transparency below.

2.2 Projection onto N and Δ States

The hedgehog form of eq. (3) obviously mixes the SU(2) space with the space of three-dimensional rotations, and Adkins, Nappi, and Witten[13], in an important paper for modern applications of skyrmions, offered a projection method for generating states of good spin and isospin. This method is very reminiscent of projection techniques long used in nuclear physics[14] to reach states of good quantum numbers from various sorts of mixed states. The general idea is to rotate the hedgehog solution using a time-dependent unitary SU(2) matrix $A(t)$ according to

$$U(\mathbf{r},\,t) = A(t)\,U_0(\mathbf{r})\,A^\dagger(t). \tag{6}$$

This matrix $A(t)$ is then converted into a dynamical variable and subjected to first-quantization, leading to a dynamic lagrangian, upon substitution into eq. (1), of the form

$$L = \int \mathcal{L}\,d\mathbf{r} = -M + I\,\mathrm{tr}\left[\frac{dA}{dt}\frac{dA^\dagger}{dt}\right], \tag{7}$$

where M is the mass defined in eq. (4) and

$$I = \frac{2\pi}{3e^3F_\pi}\int_0^\infty \sin^2 F\left\{1 + 4\left[F'^2 + \frac{\sin^2 F}{\tilde{r}^2}\right]\right\}\tilde{r}^2\,d\tilde{r}, \tag{8}$$

with the numerical result $I = 50.9\,(2\pi/3e^3F_\pi)$. The quantization procedure allows the construction of operators for spin and isospin and of wave functions in the A-space for nucleon and Δ states. Ultimately the nucleon and delta masses are given by

$$M_N = M + \frac{3}{8I}, \qquad M_\Delta = M + \frac{15}{8I}. \tag{9}$$

Table 1: Skyrmion results for $B = 1$.

Physical quantity	Theory	Experiment
F_π	108 MeV	186 MeV
$r_{\rm rms}$(ch, $T = 0$)	0.68 fm	0.72 fm
$r_{\rm rms}$(ch, $T = 1$)	1.04 fm	0.88 fm
$r_{\rm rms}$(mag, $T = 0$)	0.95 fm	0.81 fm
$r_{\rm rms}$(mag, $T = 1$)	1.04 fm	0.81 fm
μ_p	1.97 μ_0	2.79 μ_0
μ_n	-1.24 μ_0	-1.91 μ_0
g_A	0.65	1.25
$g_{\pi NN}$	11.9	13.5
$g_{\pi N\Delta}$	17.8	20.3
$\mu_{N\Delta}$	2.3	3.3

The two parameters in eq. (1) may then be fit to these two values. Higher states with $J = T$ are assumed to be nonphysical (or very broad), as in nuclear rotational bands that are truncated at the maximally allowed spin values for the number of nucleons present [14]; that role is here played by the three valence quarks. All relevant observables for the nucleon and the Δ and transitions between them can then be constructed and calculated,[13] with the rather satisfactory results (to within $1/N_c \sim 33\%$) shown in Table 1.

3 The Product Ansatz for $B = 2$ Systems

In approaching the two-nucleon problem, the earliest [12] and simplest line of attack was based on London–Heitler methods and uses the so-called product ansatz in which the two-nucleon skyrmion is assumed to be well approximated by the product of two one-nucleon forms,

$$U_{B=2}(\mathbf{r}, \mathbf{R}) = U_{B=1}(\mathbf{r} + \frac{1}{2}\mathbf{R})\, U_{B=1}(\mathbf{r} - \frac{1}{2}\mathbf{R}), \tag{10}$$

with profile functions known from the $B = 1$ problem. Here $\mathbf{r}$ is the general skyrmion variable and $\mathbf{R}$ is the separation between the two baryon centers. This ansatz automatically fulfills the correct baryon number condition $B = 2$.

It is crucial for the study of the NN system that we project the product ansatz onto nucleon states. This is accomplished by rotating each member of the product ansatz separately according to

$$U_{B=2}(\mathbf{r},\mathbf{R}) = \left[A\,U_{B=1}(\mathbf{r}+\frac{1}{2}\mathbf{R})\,A^{\dagger}\right]\,\left[B\,U_{B=1}(\mathbf{r}-\frac{1}{2}\mathbf{R})\,B^{\dagger}\right], \qquad (11)$$

where A and B are the rotation matrices of eq. (6).

4 Skyrmion Applications in Nuclei: Color Transparency

Color transparency is the phenomenon in which a nucleon is ejected from the nucleus with very high momentum transfer and experiences little final-state interaction as it exits: The high momentum transfer "catches" the nucleon in a small configuration, and, because the color charges of a small QCD object partially neutralize each other, the nucleon emerges while interacting weakly with the other nucleons. Thus color dynamics makes the nucleus transparent to the "small" nucleon (referred to here as a minisculon m). The crucial QCD ingredient in this process is precisely the mutual color neutralization, and thus it is of interest to examine this in QCD-based models. The skyrmion allows one to study this [8] by artificially shrinking the skyrmion and examining the consequent mN potential and total mN cross section. The shrinking is accomplished by using the scaling noted below eq. (5) and taking values for the minisculon such that its parameter $(eF_{\pi})_m$ becomes large. The ratio of minisculon size to the size of a normal nucleon is then

$$x \equiv \frac{r_m}{r_N} = \frac{eF_{\pi}}{(eF_{\pi})_m}. \qquad (12)$$

The result of the skyrmion calculation using the product ansatz is that the central potential tends to zero more or less linearly with x for $x \leq 0.3$. On the other hand, the range of the potential tends to fall very gradually until $x \sim 0.05$ and then plunges rapidly to zero. The total NN cross section, in a rough high-energy approximation, falls linearly to zero as x drops from unity to zero, showing quadratic behavior only when x is below 0.3. This last feature given by the skyrmion estimate is somewhat surprising in that most discussions of color transparency have based themselves on an assumption of quadratic dependence on minisculon size through the whole scale from normal radii to very small ones.

5 Skyrmion Applications in Nuclei: Shrinking and Swelling Nucleons in Nuclei

In order to make any sort of sensible estimate of nucleon swelling in nuclei, one needs to have an NN force that provides for central attraction. Among the many suggestions [1] for achieving this within the product ansatz, two seem to have provided central attraction based on physically plausible mechanisms. These are (i) the admixture of a nucleon vibrational state [15] (the Roper resonance), which appears to be the equivalent for the product ansatz of skyrmion distortions seen within more complete approaches [16]; and (ii) the inclusion of the dilaton, [17] which mocks up the trace anomaly of QCD for theories based on meson degrees of freedom.

Attraction from the Roper is usually discussed in terms of a lagrangian involving terms beyond the simple for of eq. (1). One often adds [1] terms involving four derivatives, for enhanced attraction, and terms with six derivatives, simulating ω-exchange, which are repulsive. The two-baryon states are then taken as

$$\begin{aligned} |\widetilde{NN}(R)\rangle &= \alpha(R)|NN(R)\rangle + \beta(R)|N\Delta(R)\rangle + \gamma(R)|\Delta\Delta(R)\rangle \\ &+ \epsilon(R)|NN^*(R)\rangle + \zeta(R)|N^*N^*(R)\rangle. \end{aligned} \tag{13}$$

The Roper states R^* are handled by first-quantizing a collective, breathing-mode, radial-scaling variable [1,15].

The dilaton yields attraction by sharpening the edge of the skyrmion, a reasonable physical expectation here. The dilaton itself is motivated by the need to make the lagrangian obey the same breaking of scale invariance as found in QCD in the context of the trace anomaly [17],

$$T^\mu_\mu = \partial_\mu D^\mu = -\frac{9\alpha_s}{8\pi} G^a_{\mu\nu} G^{a\mu\nu} \equiv \sigma^4, \tag{14}$$

where $D^\mu (= T^{\mu\nu} x_\nu)$ is the dilatation current, α_s is the QCD coupling constant, $G^a_{\mu\nu}$ is the gluon field, and σ is an order-parameter field—the dilaton—which represents the scalar glueball formed from the contraction of the two gluon fields. The modified skyrmion lagrangian that satisfies the trace anomaly is then

$$\begin{aligned} \mathcal{L} &= \mathrm{e}^{2\sigma}\left[\frac{1}{2}\Gamma_0^2\,\partial_\mu\sigma\,\partial^\mu\sigma - \frac{F_\pi^2}{16}\mathrm{tr}(L_\mu L^\mu)\right] + \frac{1}{32e^2}\mathrm{tr}[L_\mu, L_\nu]^2 \\ &- \frac{C_G}{4}[1 + \mathrm{e}^{4\sigma}(4\sigma - 1)]. \end{aligned} \tag{15}$$

The two parameters Γ_0 and C_G are related to the glueball—or scalar field—mass through $m_\sigma^2 = 4C_G/\Gamma_0^2$. In using the dilaton for $B = 2$ systems, one must

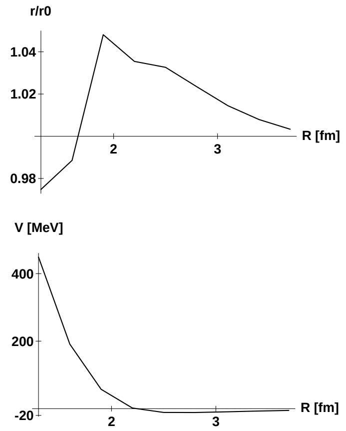

Figure 1: The NN potential and ratio of the radius for the interacting nucleon to that of the free nucleon r/r_0 for the usual skyrmion with product ansatz and baryon-resonance admixtures.

augment the product ansatz for the skyrmion by an assumption of additivity for the dilaton, $\sigma_{B=2} = \sigma_1 + \sigma_2$.

Both mechanisms are capable of yielding central attraction on the scale of 10 MeV or more with reasonable parameters. This in turn leads to nucleon swelling of some 3 or 4 percent when the separation between the nucleons places them in the attractive region. This is illustrated for the case of the Roper mechanism in fig. 1, the case for dilaton-induced attraction being quite similar [9].

6 Outlook

The two studies we have discussed represent an early phase of the applications of skyrmions to nucleon behavior and changes in the nuclear medium (see also ref. [18] in this regard). Much more can clearly be done. For example, in the context of heavy-ion collisions there is a great deal to be learned about baryons in a hot medium and here again the skyrmion may offer useful insights. We look forward to learning about these matters from experimental and theoretical work over the next decade.

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