The nucleon-nucleon force from skyrmions*

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Abstract

The applications of skyrmions to the derivation of the nucleon-nucleon force is reviewed with attention to the use of the product ansatz, additional terms in the lagrangian, baryon resonance admixtures, the instanton ansatz, dilatons, and exact two- or three-dimensional solutions for the $B = 2$ system in order to find the sources of attraction in the central and spin-orbit potentials.

1 Introduction: the application of skyrmions in nuclear physics

The original idea of Skyrme [1] to use topological solitons to describe the nucleon lay dormant for some twenty years or more and was revitalized when it became apparent that it might provide an approximate description of hadronic physics as derived from quantum chromodynamics (QCD). This came about because of the realization that in the limit of a large number of colors QCD can be reduced to a theory involving meson fields as the relevant degrees of freedom [2] and baryons can then be constructed out of these degrees of freedom as topological solitons [3]. This history is discussed in early reviews[1] on skyrmions [4–6]. Following this renewed interest a great deal of effort has gone into the study of single-baryon behavior based on the skyrmion [7], reviewed in [4–6].

From the time of Skyrme's original work [1] there has been interest in applying his model to systems with more than one nucleon. Indeed the possibility of such applications holds out hope for theories of nuclei that have their origins relatively close to the fundamental description of the strong interactions using QCD. This will be very difficult to achieve working directly from QCD because the pion is an intrinsic ingredient of low- and medium-energy nuclear physics [8, 9], yet is not easily treated within the fundamental theory or in many models inspired by it. The skyrmion is an exception in that it is automatically couched in terms of pion degrees of freedom.

Clearly before one can consider more elaborate nuclear applications of the skyrmion it is necessary to see if this approach can account for the nucleon-nucleon force. Early work on this problem [1, 10, 11],

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[1]Since there are many reviews of skyrmions and their applications, we shall here refer to original papers primarily when they address direct treatments of the two-baryon system.

reviewed in [12–19], was based on a method closely linked to the usual London–Heitler approach to the calculation of molecular potentials: it used an ansatz for the two-nucleon system that took the product of two single-nucleon fields in order to describe the $B = 2$ system. The hope was that this simple form might suffice to extract the main qualitative features of the NN potential. To some degree success was guaranteed in that the skyrmion gave reasonable values for the πNN coupling constant [7], and thus handled well the one-pion-exchange potential (OPEP) which is the key ingredient in deuteron properties, say. However, in more complex nuclei, binding is provided by the attractive isoscalar central part of the NN potential, a feature sadly lacking in the early skyrmion calculations [10, 11]. This failure may not be surprising in view of the fact that this central attraction is a very subtle feature of hadronic interactions—perhaps one of the main lessons to be learned from its exploration with various QCD-inspired models. Its depth in modern phenomenological fits is on the scale of 10 to 20 MeV, while any basic model of hadronic physics that is applied to two-nucleon systems is directed at an energy scale 100 times that much. Thus it would be surprising if anything at the level of quantitative agreement could be achieved. The main study of the central nucleon-nucleon potential in the context of skyrmions thus quickly moved to a search for mechanisms that could offer a qualitative explanation of NN central attraction. One then hopes that this will lead to new insights into general features of baryon-baryon interactions, and, ultimately, to a deeper understanding of how QCD brings about the hadronic structures we encounter: the deuteron, other bound states with two or three or four baryons in them, and heavier nuclei as well.

That program has now been underway for about a decade and we shall attempt to assess in greater detail below what progress has been made and what can be expected for the future. Certainly, in broad summary, the efforts in this field have yielded a number of possible candidates for mechanisms providing central attraction, and this has emboldened the practitioners to branch out to more intricate manifestations of two-baryon forces—for example, the NN spin-orbit forces, where attraction is again elusive—and to more complicated systems, such as $B = 2$ systems containing hyperons, $B = 3$ and 4 systems, nuclear exchange currents, and so forth. Applications of skyrmions have also been made to the study of mutual distortions in NN interactions and to other formal issues reflecting on how nucleons may behave in nuclear matter. Most of our effort here will go into surveying, as broadly as we can, the current status of our understanding of two-nucleon interactions, with some digression to related topics with direct impact on that question.

2 Skyrmion generalities

2.1 The Skyrme lagrangian and the hedgehog ansatz

The lagrangian density originally proposed by Skyrme [1] (see also [7]) for the depiction of the nucleon through a solution in terms of meson fields with nontrivial topological properties is

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 = -\frac{F_\pi^2}{16}\mathrm{tr}(L_\mu L^\mu) + \frac{1}{32e^2}\mathrm{tr}[L_\mu, L_\nu]^2, \tag{1}$$

where F_π is the pion decay constant, with experimental value 186 MeV, and e is the Skyrme parameter which accompanies his stabilizing term with four derivatives. Here

$$L_\mu \equiv U^\dagger \partial_\mu U, \tag{2}$$

where $U(\mathbf{r}, t)$ is the chiral field, at this stage taken in SU(2). For the $B = 1$ static problem we take a time-independent hedgehog solution,

$$U(\mathbf{r}, t) = \exp[i\boldsymbol{\tau}\cdot\hat{\mathbf{r}}F(r)], \tag{3}$$

where $F(r)$ is referred to as the profile function.

The strength of the topological solution for baryons in terms of meson fields lies in the realization that there exists a topological constant determined by the structure of the solution for U rather than from the specifics of the dynamical solutions to the differential equation for it. The quantity in question is defined as

$$B^{\mu} = \frac{\epsilon^{\mu\alpha\beta\gamma}}{24\pi^2} \mathrm{tr}\left[\left(U^{\dagger}\partial_{\alpha}U\right)\left(U^{\dagger}\partial_{\beta}U\right)\left(U^{\dagger}\partial_{\gamma}U\right)\right], \tag{4}$$

where $\epsilon^{\mu\alpha\beta\gamma}$ is the totally antisymmetric tensor density, and it is easily seen that

$$\partial_{\mu} B^{\mu} = 0. \tag{5}$$

Thus B^{μ} is conserved without regard to the equations of motion (i.e., it is not a Noether current) and is therefore identified as the baryon current, the corresponding baryon charge then being unconditionally conserved.

2.2 The mass spectrum and one-baryon observables

Thus far time dependence has been ignored and there has been no attempt to distinguish different baryon states through their different internal quantum numbers (spin and isospin in the present SU(2) case). Adkins, Nappi, and Witten [7] dealt with these issues by proposing a technique to rotate the hedgehog solution and project onto states of good baryon spin and isospin. To do this, they introduce time dependence through a unitary transformation

$$U(\mathbf{r},\, t) = A(t)\, U_0(\mathbf{r})\, A^{\dagger}(t), \tag{6}$$

where $A(t)$ is a unitary SU(2) matrix which is converted into a dynamical variable and quantized. By rotating the hedgehog form of eq. (3) in configuration space and in the SU(2) space one arrives at expressions [7] for the spin and isospin operators for the baryon. Eigenstates of these operators may then be constructed in the space of the parameters of the matrices A, and projection onto these states is made in the form of expectation values with respect to the eigenstates, the relevant integration being carried out over a unit sphere in four dimensions.

Last, one-baryon observables are extracted from the Noether currents generated by charge variations $\delta U = iQU$, where Q is the charge operator. Results for the root-mean-square isoscalar and isovector charge and magnetic radii, proton and neutron magnetic moments, axial coupling constant g_A, πNN coupling constant, $\pi N\Delta$ coupling constant, and $N\Delta$ transition magnetic moment are extensively reviewed elsewhere [4, 5, 6, 7], and generally agree with experiment to within about the $1/N_c \sim 33\%$ error inherent in the skyrmion.

2.3 Extensions of the lagrangian

It was early realized [20] that one could stabilize the skyrmion with terms representing ω-exchange in addition to or in place of the four-derivative form $\mathcal{L}_4$ of eq. (1) originally introduced by Skyrme. These are most easily taken in the limit of infinite ω mass as terms in the lagrangian density with six derivatives [21] in the form

$$\mathcal{L}_6 \equiv -\frac{\epsilon_6^2}{2} B_{\mu} B^{\mu}, \tag{7}$$

There are also alternative terms at the level of four derivatives, such as

$$\mathcal{L}_{4s} \equiv \frac{\gamma}{8e^2}\left[\mathrm{tr}\left(\partial^{\mu}U\,\partial_{\mu}U^{\dagger}\right)\right]^2, \tag{8}$$

where γ is again a new phenomenological parameter. This expression introduces terms with four time derivatives into the lagrangian, thus confusing the usual quantization procedures. It also tends to destabilize the skyrmion if its coefficient γ is taken large enough so as to overwhelm the other, repulsive terms $\mathcal{L}_4$ or $\mathcal{L}_6$. We thus entertain the expression of eq. (8) here reluctantly as perhaps having some limited phenomenological purpose. For the discussion of NN forces it is of some interest as possible sources of additional central attraction.

2.4 Ambiguities, parameter fitting, Casimir energy, etc.

From the early uses of the skyrmion for single-baryon properties one finds, in broad outline, two somewhat different approaches to the determination of parameters: The first of these [7] uses the baryon mass spectrum to fix the coefficients of the various terms in the lagrangian. This usually leads to overly small values for the axial coupling constant g_A, for example, but this is partly to be expected in a theory that from the very beginning is prepared to tolerate inaccuracies of about 30 percent because of the assumption that $1/N_c = 1/3$ is a large number. The second approach, represented by ref. [22] say, attempts to fit various measurable baryon matrix elements—the so-called "observables"—letting the masses come out where they will. Since the agreement with the observables is bound to be partial at best, and since refined fitting procedures make little sense in a theory not expected to work to better than 30 percent, this method is also intrinsically limited.

The situation is made more complicated by uncertain features in the calculation of baryon masses. At the level of SU(3) it proves necessary to make a large subtraction ($\sim$ 500 to 1000 MeV) of a state with the quantum numbers of the vacuum [23], so that in effect one is looking at mass splittings rather than at absolute positions. In a somewhat different approach [24], the need for an even larger subtraction is justified in terms of ambiguities in operator ordering within the lagrangian that can express themselves as a large constant term in the energy. And of course there is the question of the skyrmion Casimir energy, which is estimated [25] to be quite large ($\sim$ 1000 MeV, again with large uncertainties).

Thus there seems to be no possible hard and fast rule for fixing skyrmion parameters. In practice one is more or less limited to using a mixture of criteria based on masses and observables. The extreme cases that are weighted strongly in one direction or the other can then be seen as a measure of the inaccuracy of the overall approach.

3 Skyrmions for the NN system with the product ansatz

In order to set sensible goals for the skyrmion as applied to two-nucleon systems, it is important to keep in mind the advantages and the intrinsic limitations of the skyrmion approach as well as the overall expectations for our understanding of the nature of the nucleon-nucleon force. As we have just stressed, the skyrmion is not expected to be reliable at a level much better than 30 percent, which certainly rules it out as a method for precision calculation of NN potentials for confrontation with experimental data. This restriction is made more acute by two further concerns: The skyrmion is basically directed at the energy scale of baryons, say 1000 MeV, whereas the NN force involves features that are typically one to two orders of magnitude smaller in energy and thus may well arise from differences between much larger energy quantities, resulting in a loss of precision in the calculation. In addition, the skyrmion is a low-energy theory; it involves an expansion in derivatives, or momenta, along the lines of that used in chiral perturbation theory. Thus the short-range features of the nucleon-nucleon interaction will surely not be well represented by it. From the beginning of the study of the application of the skyrmion to NN systems, we must look for a more or less qualitative understanding of the broad features of the NN force, and we must keep in mind that the method may ultimately fail at short ranges.

On the other hand, since the skyrmion provides a reasonable representation of πNN and $\pi N\Delta$ vertices while retaining strong roots in our fundamental understanding of strong interactions through QCD, it is in some ways an ideal tool for application to nuclear physics, where pion effects are often dominant [8, 9] and quark features must clearly also be understood. In this sense it is the natural successor to the chiral bag model [26], which attempted to incorporate these same elements in nuclear applications.

3.1 The product ansatz

From the very first stage of the introduction of his topological soliton, Skyrme was interested [1] in possible applications to the two-nucleon system. Towards this end he suggested the use of the product ansatz, in which the two-baryon system is represented by a topological field which is the product of two single-baryon fields,

$$U_{B=2}(\mathbf{r},\mathbf{R}) = U_{B=1}(\mathbf{r}+\frac{1}{2}\mathbf{R})\, U_{B=1}(\mathbf{r}-\frac{1}{2}\mathbf{R}), \tag{9}$$

where $\mathbf{r}$ is the general skyrmion variable and $\mathbf{R}$ is the separation between the two baryon centers. This ansatz has the virtues that it automatically fulfills the correct baryon number condition $B = 2$ and has well-defined baryon centers by its very construction, in contrast to other approaches we shall encounter below. It also allows for simple physical interpretation when applied in the sense of a London–Heitler calculation in $B = 2$ systems.

It is crucial for the study of the NN system that we project the product ansatz onto nucleon states, if for no other reason that the dynamic admixture of Δs is often adduced as an important ingredient in providing central NN attraction, and thus we must be able to separate the nucleon from the Δ in addressing the NN force. This projection is accomplished by rotating each member of the product ansatz according to

$$U_{B=2}(\mathbf{r},\mathbf{R}) = \left[A\, U_{B=1}(\mathbf{r}+\frac{1}{2}\mathbf{R})\, A^\dagger\right]\left[B\, U_{B=1}(\mathbf{r}-\frac{1}{2}\mathbf{R})\, B^\dagger\right], \tag{10}$$

where A and B are the rotation matrices of eq. (6).

The product ansatz may now be used [10, 11] in a London–Heitler technique by choosing a separation distance $\mathbf{R}$ and calculating the resulting two-nucleon energy from the lagrangian, subtracting the masses of the two nucleons (i.e., the energy at separation $\mathbf{R} \to \infty$). This requires the evaluation of integrals over the variable $\mathbf{r}$ and averaging over the rotation variables in expectation values formed with the nucleon wave functions. Details are provided in the review of ref. [12]. At this level, and using the basic lagrangian of eq. (1), the skyrmion yields three components of the NN potential, namely,

$$V(\mathbf{R}) = V_C(R) + \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left[\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\, V_{SS}(R) + S_{12}\, V_T(R)\right], \tag{11}$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ are the Pauli spin and isospin operators and S_{12} is the usual tensor operator. The three components of the force of eq. (11) are the central part $V_C(R)$, the spin-spin part $V_{SS}(R)$, and the tensor piece $V_T(R)$. These last two are dominated by OPE, and so should be well described by the skyrmion, as indeed they prove to be [12] when compared with phenomenological spin-spin and tensor components of the NN force. The central potential is highly repulsive in its behavior for $R < 1$ fm, but no trace of attraction is found in that component for 1 fm $< R <$ 2 fm, where one knows it must occur from phenomenological potentials. In some sense it appears that the repulsion introduced through $\mathcal{L}_4$ in eq. (1) to stabilize the skyrmion spreads beyond the 1 fm range and overcomes any possible attraction. This occurs because the lagrangian is governed by two parameters which must account for baryon masses on the scale of 1000 MeV and baryon sizes on the scale of 1 fm. The smooth tail of the baryon profile then carries sizable repulsion out to the 2 fm range.

3.2 Higher-order derivatives in the lagrangian

In subsection 2.3 we noted that the symmetric fourth-order term $\mathcal{L}_{4s}$ of eq. (8) is attractive. One may thus incorporate the attractive $\mathcal{L}_{4s}$ along with the repulsive $\mathcal{L}_6$ of eq. (7) and attempt to play these off against each other in order to obtain attraction at medium NN separations while still keeping a stable skyrmion [12]. This remains an important auxiliary ingredient in various other devices for producing attraction, but does not [15, 28], in and of itself, yield attraction in $V_C(R)$, even after extensive search in the space of lagrangian parameters.

One could of course extend the skyrmion lagrangian to include the exchange of a wide variety of mesons, thus returning the problem—at least insofar as its NN applications are concerned—to the general structure of one-boson potentials. This has been considered by the Orsay group [12, 29] and by the Syracuse group [30, 31]. In an approach that is related to the inclusion of a variety of exchanged mesons, one may consider [32, 33, 34] pion quantal fluctuations about the classical skyrmion solution. This should allow for the inclusion of two-pion exchange effects that have long been known to be significant sources of central attraction, and indeed it does lead to components with attraction.

However these do not readily overcome the repulsion that still persists at 2 fm within the product ansatz at least. Thus, while this mechanism is undoubtedly of continuing importance in the skyrmion context as well, it does not seem that it solves the overall problem of central attraction within that approach.

3.3 Baryon resonance admixtures

It has long been accepted wisdom that the attraction in the central NN potential represented by so-called σ-exchange in fact arises because of two-pion exchanges which can simulate the σ effects. It is essential that in the application of box graphs of two-pion exchange to the construction of the NN potential at least one of the intermediate baryon lines refer to excited states and not to the nucleon itself. This is because in the use of the Schrödinger equation to solve the two-nucleon problem one includes all iterations of graphs with two-nucleon lines, so that the construction of the potential itself must omit these in order to avoid double counting. The physical picture is then that at intermediate ranges the two-nucleon system is partly an $N\Delta$ or $\Delta\Delta$ system or indeed some other $B = 2$ system containing higher excitations of the nucleon. This suggests that in using the skyrmion for the derivation of the NN potential it is appropriate to consider the admixture of $N\Delta$ and $\Delta\Delta$ states [28] into the wave function,

$$|\Psi(R)\rangle = \alpha(R)|NN(R)\rangle + \beta(R)|N\Delta(R)\rangle + \gamma(R)|\Delta\Delta(R)\rangle. \qquad (12)$$

Here we once again have in mind the use of the London–Heitler technique. That is to say, the calculation [28] proceeds by fixing the internucleon separation distance R and then minimizing the system energy with respect to changes in the admixture coefficients $\alpha(R)$, $\beta(R)$, and $\gamma(R)$ at the value of R in question. For this mechanism to yield appreciable intermediate-range attraction, it is necessary [35] to treat carefully the action of the tensor force in admixing spin states. Even then only very weak attraction ($\sim$ 2 to 3 MeV)—and that only for extreme skyrmion parameters which tend to overaccentuate the role of the Δ—is achieved [35, 36].

The picture in this regard is improved considerably if one entertains the admixture of the Roper $N(1440)$ resonance by extending eq. (12) to include these. The initial motivation to consider this mechanism for central attraction in the NN force arose [37] from the realization that a similar device appears importantly in deriving central attraction in the nonrelativistic quark model, where it was found [27] that the internal excitation of the individual three-quark clusters making up each nucleon had to be introduced in order to obtain a reasonable result for $V_C(R)$. The description of the Roper excitation as a vibrational state was given early on by a number of authors of which we take as representative [38]. When the vibrational problem is then solved, eigenvalues with lowest principal quantum number $n = 0$ correspond to the nucleon and Δ and those with $n = 1$ correspond to the Roper $N(1440)$ and the Δ-like particle built upon it, $\Delta(1670)$.

With reasonable parameters, the extension of eq. (12) to include the admixture of these two states then yields about 5 MeV of attraction in $V_C(R)$, which is a not unreasonable well depth given the fits of modern NN potentials such as that of Paris [39] or of Argonne [40] with well depths of 5 to 20 MeV. This depth may also be further enhanced by the correction factor for finite N_c [10], namely $((N_c + 2)/N_c)^2 = 25/9$. There is also a certain satisfaction in the fact that the skyrmion and the nonrelativistic quark model find that the same basic feature yields attraction, namely, the internal excitation of each baryon participating in the interaction between the two. This qualitative feature drives home the intrinsic subtlety of the origins of central attraction in two-baryon systems. It also hints at a finding of the exact skyrmion calculations (section 6): there too the departure of the skyrmion at short internucleon separations from its structure at $R \to \infty$ is what yields attraction, so that at least at a qualitative level many of the different approaches may in fact be convergent.

4 The instanton ansatz

It was argued by Manton [42] and then more concretely by Atiyah and Manton [43] that special care needs to be taken in generating skyrmion solutions for the NN system in order to ensure that they pertain to a manifold that includes well-separated skyrmions with arbitrary SU(2) orientations. The possibility of infinite separation is important so that one can handle the problem of removing the two baryons far away from each other, as is inherent in a scattering situation or in the definition of a potential. The freedom to orient the skyrmions in arbitrary directions is necessary so that one can project onto states with nucleon quantum numbers. To describe two well-separated skyrmions with independent positions and orientations requires 12 degrees of freedom. The product ansatz satisfies this requirement, of course, and so easily succeeds in giving a reasonable asymptotic description of the NN system, but its "product" nature is overly restrictive, as we have seen in the previous section, and thus fails at small distances. Other complete solutions do not always fulfill these conditions and thus may lead to descriptions of $B = 2$ systems with unphysical properties, as we shall discuss further in section 6. This led to a search for skyrmion ansätze for the $B = 2$ system which possess these two essential requirements while still allowing for relatively simple treatment.

The full implementation of this program for the calculation of the NN potential was carried out ([44, 45], reviewed in [18]; see also [46] and applications to systems with $B = 3$, 4 in [47]). The procedure produces excellent results [45] for the OPEP-dominated spin-spin and tensor parts of the NN force, but ultimately fails to find attraction for the central part of the potential. Indeed, the results for $V_C(R)$ are not very different from those obtained using the original product ansatz, which thus finds some substantiation in the instanton ansatz.

5 The dilaton

It is well known that QCD with massless quarks is scale invariant when treated as a classical theory and loses this property upon quantization. Accompanying this is the trace anomaly, whereby the trace of the energy-momentum tensor $T^{\mu\nu}$, which vanishes classically, is given after quantization by

$$T^{\mu}_{\mu} = \partial_{\mu} D^{\mu} = -\frac{9\alpha_s}{8\pi} G^a_{\mu\nu} G^{a\mu\nu} \equiv \psi^4, \tag{13}$$

where $D^{\mu}(= T^{\mu\nu} x_{\nu})$ is the dilatation current, α_s is the QCD coupling constant, $G^a_{\mu\nu}$ is the gluon field, and ψ is an order-parameter field—the dilaton—which represents the scalar glueball formed from the contraction of the two gluon fields. It was early pointed out that an effective lagrangian based on this scalar field can be used to incorporate the trace anomaly [48], and the consequences of such an approach for scalar glueballs were studied some time thereafter [49], along with modifications to the Skyrme lagrangian in order to incorporate there first the appropriate scale invariance and then the trace anomaly [50, 51]. The physical picture for the skyrmion plus dilaton is that a kind of bag is formed by the dilaton field such that the skyrmion is located within a region where the gluon condensate is suppressed. This also answers for one of the obvious limitations of the skyrmion as it stands alone, namely, that there is only one length parameter in the basic model, so that there is no well-defined concept of a "surface thickness" or baryon edge. Since the skyrmion is characterized by the lagrangian of eq. (1) in terms of the energy unit F_π/e and length unit eF_π, the need to fit single-baryon properties automatically introduces a scale of hundreds of MeV for the energy and about a fermi for sizes. Thus when we search for attraction in the NN force we must overcome the tail of the very strong repulsion at zero separation which still persists strongly in the range of 2 fm or so of relevance for attraction. Obviously, the sharpening of the baryon surface will help to reduce this problem.

Schechter [48] and his coworkers [49, 50, 51] introduced a lagrangian for the dilaton field through the consideration of the trace anomaly. If terms with no more than two time derivatives appear, the most general form allowed is

$$\mathcal{L}_{\text{dilaton}} = \mathcal{L}_{2\ \text{dilaton}} - V_{\text{dilaton}}(\psi) = \frac{1}{2} b^2 \partial_\mu \psi \partial^\mu \psi - \frac{1}{4} \psi^4 \log\left[\frac{\psi^4}{\Lambda^4}\right], \tag{14}$$

where b is a dimensionless constant and Λ is a constant with dimensions of energy. For the mean-field case, eq. (14) then leads to $\langle\psi\rangle = \Lambda/\mathrm{e}^{1/4}$. Given this dilaton lagrangian, it is easy to modify [50, 51] the basic Skyrme lagrangian (1) in order to incorporate the trace anomaly. It is common practice to replace ψ with an exponentiated scalar field σ, $\psi \equiv \langle\psi\rangle\, \mathrm{e}^{\sigma}$. The overall lagrangian, including the terms with six derivatives of section 3.2, is then

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{2\ \mathrm{dilaton}} + \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 - V(\sigma) \\ &= \mathrm{e}^{2\sigma}\left[\frac{1}{2}\Gamma_0^2\,\partial_\mu\sigma\,\partial^\mu\sigma - \frac{F_\pi^2}{16}\mathrm{tr}(L_\mu L^\mu)\right] + \frac{1}{32e^2}\mathrm{tr}[L_\mu,\, L_\nu]^2 \\ &- \epsilon^2\mathrm{e}^{-2\sigma}B_\mu B^\mu - \frac{C_G}{4}[1+\mathrm{e}^{4\sigma}(4\sigma-1)]. \end{aligned} \tag{15}$$

The two parameters Γ_0 and C_G are related to the glueball—or scalar field—mass through $m_\sigma^2 = 4C_G/\Gamma_0^2$. For frequently-used values for these parameters this may yield a rather low mass for the σ glueball field, which one must assume then mixes strongly with $\pi\pi$-exchange simulations of σ-like effects; nonetheless, the virtue of the dilaton approach is that is provides a natural source for this scalar rather than lying on a completely ad hoc introduction of it into the theory. (Other criticisms of the use of the dilaton in nuclear-matter applications have been noted in ref. [52].)

Once the dilaton is coupled to the skyrmion, it provides a very natural source of central attraction [53]. The usual product ansatz must be modified by an assumption of dilaton additivity, $\sigma_{B=2} = \sigma_1 + \sigma_2$, which is assumed to be of sufficient accuracy for purposes of calculating the NN force since mixed terms are cut out in the function $\psi_{B=2} = \langle\psi_{B=2}\rangle\, \mathrm{e}^{\sigma_{B=2}}$. Indeed, ref. [53] finds some 35 MeV of attraction in the central NN potential for $r \sim 1.8$ fm. We return to the application of the dilaton to the NN potential in section 8 below, both with regard to its use for $V_C(R)$ and for the spin-orbit potential.

6 The exact skyrmion solution for $B = 2$

Recently exact numerical solutions of the full $B = 2$ skyrmion problem (on restricted manifolds) have become increasingly available. The review of Walhout and Wambach [19] devotes itself in considerable degree to this topic, in which these authors have themselves been very much involved. Early exact numerical solutions [54, 55, 56, 57] for the $B = 2$ system were obtained by relaxing the full static equations of motion on a grid within a particular geometry. These solutions have a toroidal shape, the hole in the middle being, presumably, the reflection of short-range repulsion. They do not show any residual signs of the presence of two separate nucleons, which raises the criticism noted by Manton [42] (discussed briefly in subsection 4) that the restricted manifold of this solution does not allow sufficient degrees of freedom to describe correctly the complete dissolution into two baryons.

Subsequent exact numerical studies were usually based on an assumption of adiabaticity, whereby one introduces constraints to fix the collective coordinates and solves the static equations of motion (as opposed to trying to follow the full time evolution of two skyrmions aimed at each other; see [54, 58, 59, 60]). The fixing of coordinates pertains both to the relative orientation of the skyrmions and to the distance between their centers. The latter may be fixed through Lagrange multipliers (see also [61, 62]) while the relative orientations between the two skyrmions are chosen in the same manner (subsection 4) as for the instanton solution. The solutions [19, 63] then show two separated nucleons which coalesce at about 1.2 fm for the maximally attractive relative orientation of the skyrmions, and then meld into a toroid below that "separation" distance. A very small amount of central attraction may be found in this approach; this is significantly increased by admixing [65, 66, 67] $N\Delta$, ΔN, and $\Delta\Delta$ states, as discussed in subsection 3.3. Attraction is also augmented by incorporating [64] the dilaton, in which case the "bags" provided by the dilaton merge at about 1.4 fm, and an attractive central component on the scale of 20 MeV is found. This is further enhanced by again considering the admixture of states containing Δs.

The ability to carry out these difficult calculations is encouraging, and no doubt goes far towards providing a mechanism of central attraction in two-nucleon interactions in the skyrmion approach, but more must be done to refine them in order to be sure that all artifacts of the numerical procedures have been fully eliminated [68]. This may also lead to a reliable assessment as to which ingredients are essential to obtain central attraction. The connections with others approaches should also be pursued further in the hope of eventually constructing a model simple enough for application in more complex nuclear situations.

Quite recently Leese, Manton, and Schroers [69] have produced a skyrmion solution for the deuteron carried out on a ten-dimensional manifold which yields properties close to those of the physical deuteron. It should be emphasized that this is the first case of a realistic representation of the deuteron within the skyrmion approach. As noted in section 4, the NN system should ideally be calculated on a twelve-dimensional manifold in order to include full freedom for the locations of the two skyrmions and for their SU(2) orientations. This calculation still proves prohibitively difficult, however, and so ref. [69] uses the lowest-dimensional manifold that still includes both a toroidal configuration of minimal energy and configurations that approximate the infinite separation of two skyrmions oriented in the maximally attractive relative direction in SU(2). To perform the calculation, an instanton assumption is again made (section 4) and the maximally attractive orientation—rotation by π about the axis perpendicular to the line joining the skyrmions—is taken (see the end of section 4). The geometry that underlies the instanton is somewhat complicated in order to accommodate the limiting toroidal and separated configurations. The technical aspect of the calculation then involves evaluating the lagrangian after restricting to that geometry, and generating the hamiltonian with first-quantized collective degrees of freedom. Numerical values for deuteron properties in this approach and in that of ref. [57] have been calculated, from which one sees that results from the ten-dimensional manifold M_{10} are far closer to those of experiment than are the ones arising [57] from an eight-dimensional one M_8, because the larger dimensionality is necessary in order to allow the skyrmions to sample a space that allows also for genuine separation of the two nucleon centers.

7 Systems with nonzero strangeness

As we have seen, the origins of central attraction at medium ranges in the NN force are still unclear. We have a loose understanding of how it might come about in models that base themselves on QCD, but little more than that. And the skyrmion, with perhaps the best claim amongst such models to a genuine QCD pedigree, mostly serves to stress that central attraction is a subtle affair, which may derive from several different mechanisms. Nor is it clear if the problems lie in the basic notion of the skyrmion (i.e., of the large N_c limit leading to mesonic degrees of freedom interpreted as baryons through a topological argument) or in the particular lagrangian proposed for it (i.e., might medium ranges require more field derivatives?). All of this makes it especially interesting to consider systems with nonzero strangeness, where very similar mechanisms may enter and one may be able to highlight specific features of the $B = 2$ interaction by exploiting the different quantum numbers of the $S \neq 0$ systems.

The extension to SU(3) of the previous SU(2) methods based on the product ansatz and coupling to rotational and vibrational degrees of freedom shows similar amounts of attraction as were found before [70]. This extension also allows for the study [71, 72] of ΣN, ΛN, and $\Lambda\Lambda$ interactions, to list only those for which there exist some rough data at present. Once again central attraction is found if Roper vibrations are admixed, as for the NN case. The early work [71] in this sector was based on Monte Carlo methods for the projection onto baryon states with appropriate quantum numbers, which has an advantage of algebraic simplicity and flexibility, but inevitably carries with it the the need for long computer runs and the imprecision of the statistical method. Central attraction as strong or stronger than for the NN system was found. The more accurate algebraic approach [72] found sizable attraction in systems with identical baryons, for example, NN and $\Lambda\Lambda$, when further symmetry breaking was introduced over and above that of the basic SU(3) lagrangian, i.e., when using $F_\pi \neq F_K$. This attraction increases with increasing strangeness content, so that at small interbaryon separation the central $\Xi\Xi$ potential is about four times more attractive than the NN one. The bulk of the attraction from this mechanism occurs at values of $R \leq 1$ fm, however, which reduces somewhat its relevance for conventional central baryon-baryon attraction.

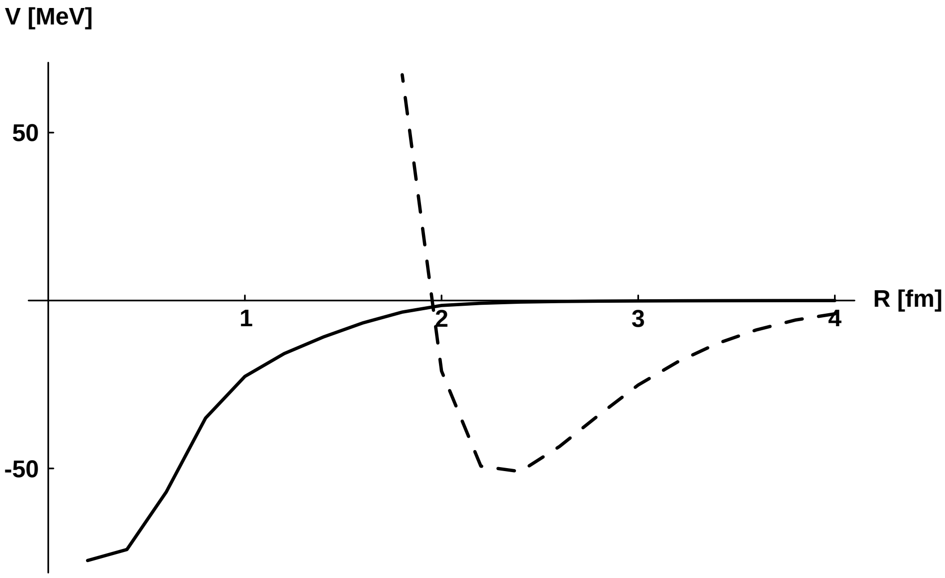

Figure 1: Spin-orbit potential and central potential (dashed line for $r \geq 2$ fm) for skyrmion parameters $F_\pi = 143$ MeV and $e = 20.0$, and dilaton parameters $\Gamma_0 = 137$ MeV and $C_G = (121 \text{ MeV})^4$, yielding a nucleon mass $M_N = 942$ MeV and a Δ mass $M_\Delta = 1174$ MeV. This result requires the presence of an $\mathcal{L}_{6,2}$ term and a dilaton.

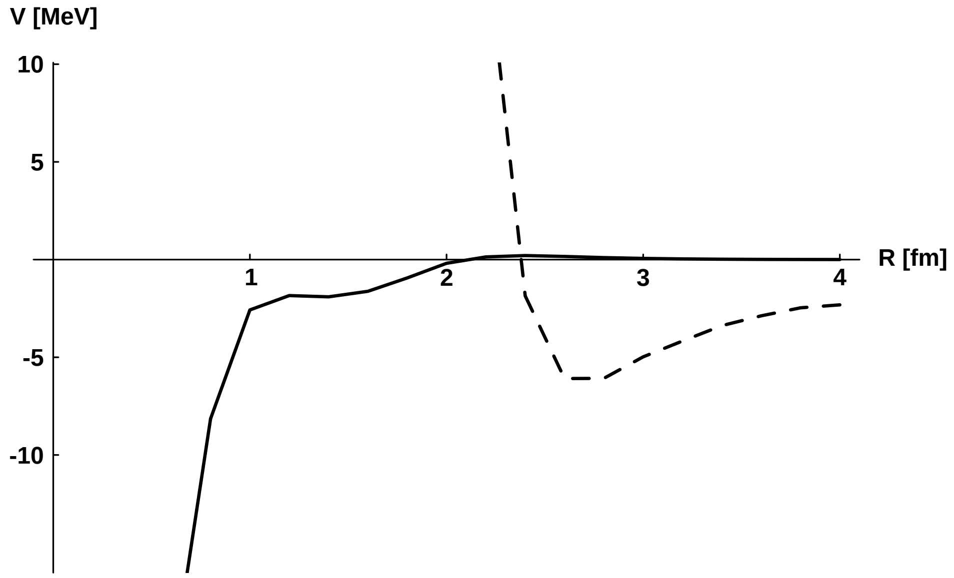

Figure 2: Spin-orbit potential and central potential as in fig. 1, but with dilaton parameters $\Gamma_0 = 61.2$ MeV and $C_G = (157.3\ \text{MeV})^4$ yielding a glueball mass of 809 MeV.

8 The spin-orbit force

The study of the NN isoscalar spin-orbit force parallels in some respects the earlier work on the isoscalar central force: Lowest-order product-ansatz results again [41, 73, 74] show the wrong sign, that is, they find repulsion instead of attraction. Riska and Schwesinger [75] then suggested that the term $\mathcal{L}_6$ of eq. (7) would yield attraction, which indeed it does, but only at the cost of an unusually large coupling constant for this term [76]. The introduction of the dilaton [75] can enhance this attraction, as has been worked out more fully [77] using the lagrangian of eq. (14). In the course of carrying out such calculations one must deal with the "dragged" skyrmion, that is, the spin-orbit force is identified through the appearance of a term involving the motion of the baryon itself which must enter in the orbital angular momentum operator $\mathbf{L} = \mathbf{R} \times \mathbf{P}$ coupled to the spin $\boldsymbol{\sigma}$. All this comes about through a collective coordinate referring to the baryon position, in addition to the collective rotation.

Choosing the dilaton parameters $\Gamma_0 = 137$ MeV and $C_G = (121\ \text{MeV})^4$, with the empirical value for the ωNN coupling constant, $g_\omega^2/4\pi \sim 10$, sizable attraction is found [77], as seen in fig. 1, both for the central and the spin-orbit isoscalar components. These dilaton parameters imply a very low value for the glueball mass (~ 214 MeV) which is inconsistent with empirical expectations. If we take $\Gamma_0 = 61.2$ MeV and $C_G = (157.3\ \text{MeV})^4$ for a higher glueball mass of 809 MeV we get the results of fig. 2, with about an order of magnitude less attraction, or about the same well depth as implied by vibrational admixtures (section 3.3). Since there are sizable technical difficulties [77, 78] in considering the moving skyrmion—both at the computational level and in matters of principle—it would be premature to argue that the spin-orbit problem is solved. Indeed in addition to this special aspect of the spin-orbit force it also suffers from all the uncertainties noted for the more "static" central force. The particular choice of dilaton parameters also leads to a very low glueball mass. Although the dilaton mechanism itself is a well-motivated ingredient of skyrmion physics by virtue of the need to encompass the QCD trace anomaly, the appropriate values for the glueball mass involved are not clear and may well have to be higher than the one used here (cp. [79]).

9 Summary

The first decade of studies of the two-baryon system with skyrmions has yielded an understanding of several different mechanisms that may contribute to central attraction and to the attractive isoscalar spin-orbit force. Thus the way is open to sort out, within the next few years, which of these effects are essential and dominant. It appears clear from QCD-related studies of the NN interaction so far that central attraction is the result of an extremely subtle play of QCD effects.

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