TWO-BARYON FORCES FROM SKYRMIONS[a]

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Abstract

Background material on solitons and, especially, skyrmions is provided and the applications of the latter to the derivation of the nucleon–nucleon force is reviewed with attention to the use of the product ansatz, additional terms in the lagrangian, baryon resonance admixtures, dilatons, and exact two- or three-dimensional solutions for the $B = 2$ system in order to find the sources of attraction in the central and spin–orbit potentials. We discuss extensions to two-baryon systems with nonzero strangeness and address applications to the behavior of the nucleon in nuclei achieved from skyrmions.

[a] Based on lectures delivered by J.M. Eisenberg at Hadron Physics 96, Brazil, April, 1996.
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1 Introduction: the application of skyrmions in nuclear physics

The original idea of Skyrme[1] to use topological solitons to describe the nucleon was resurrected some 20 years after its original introduction when it became apparent that it might provide an approximate description of hadronic physics as derived from quantum chromodynamics (QCD). This came about because of the realization that in the limit of a large number of colors QCD can be reduced to a theory involving meson fields as the relevant degrees of freedom[2] and baryons can then be constructed out of these degrees of freedom as topological solitons.[3] This history is discussed in early reviews[c] on skyrmions.[4,5,6] Following this renewed interest[7] a great deal of effort has gone into the study of single-baryon behavior based on the skyrmion,[7] reviewed in.[4,5,6]

From the time of Skyrme's original work[1] there has been interest in applying his model to systems with more than one nucleon. Indeed the possibility of such applications holds out hope for theories of nuclei that have their origins relatively close to the fundamental description of the strong interactions using QCD. This will be very difficult to achieve working directly from QCD because the pion is an intrinsic ingredient of low- and medium-energy nuclear physics[8,9] yet is not easily treated within the fundamental theory or in many models inspired by it. The skyrmion is an exception in that it is automatically couched in terms of pion degrees of freedom.

Clearly before one can consider more elaborate nuclear applications of the skyrmion it is necessary to see if this approach can account for the nucleon–nucleon force. Early work on this problem[1,10,11], reviewed in[15,16,17,18,19,20,21,22], was based on a method closely linked to the usual London–Heitler approach to the calculation of molecular potentials: it used an ansatz for the two-nucleon

[c] Since there are many reviews of skyrmions and their applications, we shall here refer to original papers primarily when they address direct treatments of the two-baryon system.

system that took the product of two single-nucleon fields in order to describe the $B = 2$ system. The hope was that this simple form might suffice to extract the main qualitative features of the NN potential. To some degree success was guaranteed in that the skyrmion gives reasonable values for the πNN coupling constant[7], and thus handles well the one-pion-exchange potential (OPEP) which is the key ingredient in deuteron properties, say. However, in more complex nuclei, binding is provided by the attractive isoscalar central part of the NN potential, a feature sadly lacking in the early skyrmion calculations[10,11]. This failure may not be surprising in view of the fact that this central attraction is a very subtle feature of hadronic interactions—perhaps one of the main lessons to be learned from its exploration with various QCD-inspired models. The central well depth in modern phenomenological fits is on the scale of 10 to 20 MeV, while any basic model of hadronic physics that is applied to two-nucleon systems is directed at an energy scale 100 times that much. Thus it would be surprising if anything at the level of quantitative agreement could be achieved. The main study of the central nucleon–nucleon potential in the context of skyrmions thus quickly moved to a search for mechanisms that could offer a qualitative explanation of NN central attraction. One then hopes that this will lead to new insights into general features of baryon–baryon interactions, and, ultimately, to a deeper understanding of how QCD brings about the hadronic structures we encounter: the deuteron, other bound states with two or three or four baryons in them, and heavier nuclei as well.

That program has now been underway for about a decade among some ten or a dozen groups around the world and we shall attempt to assess in greater detail below what progress has been made and what can be expected for the future. Certainly, in broad summary, the efforts in this field have yielded a number of possible candidates for mechanisms providing central attraction,

and this has emboldened the practitioners to branch out to more intricate manifestations of two-baryon forces—for example, the NN spin–orbit forces, where attraction is again elusive—and to more complicated systems, such as $B = 2$ systems containing hyperons, $B = 3$ and 4 systems, nuclear exchange currents, and so forth. Applications of skyrmions have also been made to the study of mutual distortions in NN interactions and to other formal issues reflecting on how nucleons may behave in nuclear matter.

Generally, our review here will be cursory where we cover material that has already been extensively reviewed. We present these items again in order to have a consistent notation and a reasonably continuous coverage of the area. Most of our effort will go into surveying, as broadly as we can, the current status of our understanding of two-nucleon interactions, with some digression to related topics with direct impact on that question.

2 Soliton generalities

2.1 Solitary waves and solitons

We include here some introductory material on solitary waves and solitons in general in order to avoid giving the impression that skyrmions came into existence out of a total void with regard to the existence of "self-standing" solutions of nonlinear wave equations representing lumps of energy isolated in space. Excellent books[12,13,14] exist in this area and should be consulted for a more complete picture.

The story starts in 1835 or so with a horseback ride of John Scott Russell, a person with much more than average knowledge of and interest in hydrodynamical problems. As he rode along a barge canal, he observed the build-up of a wave in front of a barge being dragged along the canal. When the barge

stopped, this wave broke away and traveled on its own for several kilometers without change in form or velocity. These features are what characterize a solitary wave, as first studied by Russell after that event: the solitary wave is a localized lump of energy that retains its shape and velocity as it propagates. If, in addition, the solitary wave is unchanged upon collision with other solitary waves, then it is referred to as a soliton. Our own interest is in a particular soliton called the skyrmion.

2.2 Solutions in $d = 1 + 1$

Let us consider the situation in one spatial dimension and one time dimension ($d = 1 + 1$). For the massless case $m = 0$ the wave equation reads

$$\Box\phi = \partial_\mu \partial^\mu \phi = \left[\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right]\phi(\mathbf{x}, t) = 0, \tag{1}$$

and has the well-known solution

$$f(x \pm ct); \quad \cos(kx \pm ct), \quad \sin(kx \pm ct), \tag{2}$$

where f is an arbitrary function of the particular combination $x \pm ct$ and one can also write the solution in terms of the Fourier transform

$$f(x - ct) = \int \frac{dk}{2\pi}\big[a(k)\cos(kx - \omega t) + b(k)\sin(kx - \omega t)\big], \tag{3}$$

where the wave travels with frequency $\omega = kc = k$. Here $a(k)$ and $b(k)$ are arbitrary functions chosen to match initial conditions, say. The simple relation between ω and k implies that the wave for the massless case travels with a constant velocity which is the same for all components, so that the wave is undistorted.

When the mass is different from zero the wave equation becomes

$$(\Box + m^2)\phi = \left[\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} + m^2\right]\phi = 0 \tag{4}$$

and thus the relationship between frequency and wave number is $\omega^2 = k^2 + m^2$ so that different ks travel with different velocities $\omega(k)/k$ and are distorted by the propagation.

It is possible to compensate for the presence of a nonvanishing mass by introducing nonlinearities. The massless case for the so-called ϕ^4-theory has a wave equation

$$\Box\phi + \phi^3 = 0 \tag{5}$$

with energy

$$E[\phi] = \int_{-\infty}^{\infty} dx \left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\phi'^2 + \frac{1}{4}\phi^4\right]. \tag{6}$$

This is minimized by the trivial solution $\phi \equiv 0$.

When a mass-like term is introduced,

$$\ddot{\phi} - \phi'' - \phi + \phi^3 = 0, \tag{7}$$

with the corresponding energy

$$E[\phi] = \int_{-\infty}^{\infty} dx \left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\phi'^2 + \frac{1}{4}(\phi^2 - 1)^2\right]. \tag{8}$$

Here there occurs a minimum in the energy at $\phi \equiv \pm 1$, corresponding to a spontaneous symmetry breaking. The localized solution must approach ± 1 as $x \to \pm\infty$ at any t.

Generally, a *solitary wave* is defined as a localized, nonsingular solution of the nonlinear field equation. Its energy density has the form

$$\epsilon(\mathbf{x}, t) = \epsilon(\mathbf{x} - \vec{u}t), \qquad \vec{u} = \text{ velocity} = \text{ constant}. \tag{9}$$

We further define a *soliton* to be such that under mutual scattering of the solitary waves no change occurs in speed or shape, but only in the central

positions of the waves. Thus the initial energy distribution

$$\epsilon(\mathbf{x},t)_{\overrightarrow{t\to-\infty}}\sum_{i=1}^{N}\epsilon_0(\mathbf{x}-\vec{a}_i-\vec{u}_i t) \tag{10}$$

goes over after scattering has taken place to

$$\epsilon(\mathbf{x},t)_{\overrightarrow{t\to+\infty}}\sum_{i=1}^{N}\epsilon_0(\mathbf{x}-\vec{a}_i-\vec{u}_i t+\vec{\delta}_i), \tag{11}$$

where $\vec{\delta}_i$ is a constant so that the energy distributions are only shifted in location.

For $d=1+1$, we can solve the general equation for solitary waves. The lagrangian in that case is

$$\mathcal{L}=\frac{1}{2}\dot{\phi}^2+\frac{1}{2}\phi'^2-U[\phi], \tag{12}$$

and variations with respect to ϕ immediately lead to the Euler–Lagrange equations of motion

$$\delta\int dt\int_{-\infty}^{\infty}dx\Rightarrow\Box\phi=\ddot{\phi}-\phi''=-\frac{\partial U}{\partial\phi} \tag{13}$$

and energy

$$E[\phi]=\int_{-\infty}^{\infty}dx\left[\frac{1}{2}\dot{\phi}^2+\frac{1}{2}\phi'^2+U[\phi]\right]. \tag{14}$$

Note that for $d=1+1$ the wave function ϕ is dimensionless, while for $d=3+1$ it has the dimensions of energy; dimensional considerations will be of help when we come to consider dilatons below.

For the static case (i.e., no time dependence), multiplying by ϕ' immediately yields

$$\int_{-\infty}^{x}dx\phi'\phi''=\int_{-\infty}^{x}dx\frac{\partial U}{\partial\phi}\phi' \tag{15}$$

or

$$\frac{1}{2}\phi'^2=U[\phi] \tag{16}$$

since ϕ' and $U[\phi] \to 0$ as $x \to 0$. Thus $\phi' = \pm\sqrt{2U}$ and

$$x - x_0 = \pm \int_{\phi(x_0)}^{\phi(x)} \frac{d\tilde{\phi}}{\sqrt{2U[\tilde{\phi}]}} \tag{17}$$

is the general solution for the solitary wave, to be inverted to produce ϕ as a function of x.

2.3 The kink

For the ϕ^4 theory, we have

$$U = \frac{\lambda}{4}\left(\phi^2 - \frac{m^2}{\lambda}\right)^2, \qquad \lambda,\, m^2 > 0. \tag{18}$$

Then

$$\ddot{\phi} - \phi'' = m^2\phi - \lambda\phi^3 \tag{19}$$

and

$$\begin{aligned} x - x_0 &= \pm\frac{1}{\sqrt{\lambda/2}} \int_{\phi_0}^{\phi} \frac{d\tilde{\phi}}{\tilde{\phi}^2 - m^2/\lambda} \\ &= \mp\frac{\sqrt{2}}{m}\mathrm{arctanh}\frac{\sqrt{\lambda}}{m}\phi(x) \end{aligned} \tag{20}$$

or

$$\phi(x) = \mp\frac{m}{\sqrt{\lambda}}\tanh\left(\frac{m}{\sqrt{2}}(x - x_0)\right), \tag{21}$$

which is referred to as "the kink." Its energy density is

$$\epsilon(x) = \frac{1}{2}\phi'^2 + U = \frac{m^4}{2\lambda}\mathrm{sech}^4\frac{m(x - x_0)}{\sqrt{2}}. \tag{22}$$

The kink simply rises gently from $-m/\sqrt{\lambda}$ to $m/\sqrt{\lambda}$, the rise being centered (anti)symmetrically about $x = x_0$. The energy density is a lump, also centered about this value, with width $1/m$. The (finite!) classical mass is then

$$M_{\mathrm{classical}} = \int_{-\infty}^{\infty} dx\epsilon(x) = \frac{2\sqrt{2}}{3}\frac{m^3}{\lambda}. \tag{23}$$

The kink is a solitary wave but *not* a soliton. For nonzero speed the solution is

$$\phi_u(x,t) = \frac{m}{\sqrt{\lambda}} \tanh\left[\frac{m}{\sqrt{2}} \frac{x - x_0 - ut}{\sqrt{1-u^2}}\right], \qquad u \neq 0, \tag{24}$$

and the energy is

$$E = \frac{M_{\text{classical}}}{\sqrt{1-u^2}}. \tag{25}$$

Note that ϕ_u is singular in λ and so there can be no perturbative solution about $\lambda = 0$. The solution for ϕ has two degenerate minima at $\phi = \pm m/\sqrt{\lambda}$.

It is possible to define here what we shall refer to as a topological charge (with one eye on future developments of this sort): Consider the current

$$k^\mu = \frac{\sqrt{\lambda}}{m} \epsilon^{\mu\nu} \partial_\nu \phi, \qquad \left(\epsilon^{01} = -\epsilon^{10} = 1,\ \epsilon^{00} = \epsilon^{11} = 0\right). \tag{26}$$

This current is defined without reference to symmetries of the lagrangian or details of the dynamics and thus is a not a current of the variety generated by the usual considerations of Noether. This current is conserved by the intrinsic properties of its definition (thus in part justifying the appellation "topological"),

$$\partial_\mu k^\mu = 0 \qquad \left(g^{00} = -g^{11} = 1, g^{01} = g^{10} = 0\right). \tag{27}$$

The corresponding charge is

$$Q = \int_{-\infty}^{\infty} dx k^0 = \frac{\sqrt{\lambda}}{m} \int_{-\infty}^{\infty} dx \frac{\partial \phi}{\partial x} = \frac{\sqrt{\lambda}}{m} [\phi(\infty) - \phi(-\infty)], \tag{28}$$

is also conserved of course.

2.4 The sine-Gordon equation

We shall also consider briefly the sine–Gordon equation in order to show a case which is not only a solitary wave, but also a soliton. The defining equation

here is

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{m^4}{\lambda}\left(\cos\frac{\sqrt{\lambda}}{m}\phi - 1\right)$$
$$\sim \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 - \frac{\lambda^2}{6!}\frac{\phi^6}{m^2}, \tag{29}$$

where the first two terms are the usual free Klein–Gordon case. Then the Euler–Lagrange equation is

$$\Box\phi + \frac{m^3}{\sqrt{\lambda}}\sin\frac{\sqrt{\lambda}}{m}\phi = 0, \tag{30}$$

with $\bar{x} = mx,\ \bar{t} = mt$, and $\bar{\phi} = (\sqrt{\lambda}/m)\phi$. So

$$\bar{\mathcal{L}}(\bar{x},\bar{t}) = \frac{m^4}{\lambda}\left[\frac{1}{2}\bar{\partial}_\mu\bar{\phi}\bar{\partial}^\mu\bar{\phi} + \cos\bar{\phi} - 1\right]. \tag{31}$$

Dropping the bars, the equation of motion is

$$\ddot{\phi} - \phi'' + \sin\phi = 0, \tag{32}$$

with energy

$$E = \frac{m^3}{\lambda}\int dx\left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\phi'^2 + 1 - \cos\phi\right]. \tag{33}$$

Here there are discrete symmetries

$$\phi \to -\phi; \qquad \phi \to \phi + 2N\pi,\ N = \text{ positive integer}, \tag{34}$$

and the energy E vanishes at $\phi = 2N\pi$. The charge is given by

$$Q = \frac{1}{2\pi}\int_{-\infty}^{\infty} dx\frac{\partial\phi}{\partial x} = N_+ - N_-, \tag{35}$$

and the solution for ϕ is

$$x - x_0 = \pm\int_{\phi_0}^{\phi}\frac{d\tilde{\phi}}{\sqrt{2U}} = \pm\int_{\phi_0}^{\phi}\frac{d\tilde{\phi}}{2\sin(\tilde{\phi}/2)} = \pm\log\tan(\tilde{\phi}/4)\Big|_{\phi_0}^{\phi}. \tag{36}$$

Thus finally the static soliton solution is

$$\phi_s = 4\arctan\left[\exp[x - x_0]\right], \qquad Q = 1 \tag{37}$$

for the final static soliton solution.

The nonstatic version is

$$\phi_s = 4\arctan\left[\exp[\frac{x - x_0}{\sqrt{1-u^2}}]\right] \tag{38}$$

and the antisoliton is given by $\phi_a = -\phi_s, \quad Q = -1$. The soliton energy is $E = M_s = 8m^2/\lambda$.

When one considers, for example, a soliton–antisoliton pair, the relevant wave function is given by

$$\phi_{sa} = 4\arctan\frac{\sinh\frac{ut}{\sqrt{1-u^2}}}{u\cosh\frac{x}{\sqrt{1-u^2}}}, \tag{39}$$

satisfying

$$\ddot{\phi} - \phi'' + \sin\phi = 0. \tag{40}$$

After some small tedium one finds the asymptotic behavior

$$\begin{aligned}
\phi_{sa} \underset{t\to-\infty}{\longrightarrow} & \; 4\arctan\exp\left[\frac{1}{r}[x - u(t+\Delta)\right] \\
- & \; 4\arctan\exp\left[\frac{1}{r}[-x - u(t+\Delta)\right] \\
= & \; \phi_s\left(\frac{x - u(t+\Delta)}{\sqrt{1-u^2}}\right) - \phi_a\left(\frac{-x - u(t+\Delta)}{\sqrt{1-u^2}}\right),
\end{aligned} \tag{41}$$

where

$$r \equiv \sqrt{1-u^2} \quad \text{and} \quad \Delta \equiv \frac{\sqrt{1-u^2}}{u}\log u < 0 \quad \text{for } u < 1. \tag{42}$$

Similarly,

$$\phi_{sa} \underset{t\to+\infty}{\longrightarrow} \phi_s\left(\frac{x - u(t-\Delta)}{\sqrt{1-u^2}}\right) - \phi_a\left(\frac{-x - u(t-\Delta)}{\sqrt{1-u^2}}\right), \tag{43}$$

so that the effect of a scattering is a shift by 2Δ, but no change in speed or shape—the characteristics of a soliton.

3 The $N_c \to \infty$ limit

In the modern context, the skyrmion is important as a limiting solution for QCD, namely, one that applies when $N_c \to \infty$. As we noted, 't Hooft[2] showed that QCD could be treated purely in terms of meson degrees of freedom in the sector with no baryons, and Witten[3] extended this to $B \neq 0$ by considering a topological solution for the meson fields. A natural candidate for the lagrangian of these fields was then that of Skyrme[1].

A concise pedagogical discussion of the $N_c \to \infty$ limit is given in the book of Bhaduri[23], and this is summarized here only in the briefest of terms: The general idea is to take the coupling constant g of QCD to be proportional to $1/\sqrt{N_c}$ so that $g^2 N_c$ remains constant when N_c grows. This insures that the one-gluon-loop insertion into a gluon line remains finite (in fact of order N_c^0) when $N_c \to \infty$. This is because the coupling constants yield a factor $1/N_c$ while the number of possibilities for internal gluons is $\propto N_c$. The single-quark-loop insertion into the gluon line, on the other hand, is of order $1/N_c$, since there is only one possibility for an internal quark pair once the gluon is fixed. In general, examination of planar graphs—those in which all crossed gluons interact, so that one need not imagine a second plane for a crossed gluon to move on—shows them to be of order N_c^0, while nonplanar graphs are of order $1/N_c^2$. This is again brought about by the number of internal possibilities within the loops in question.

One can analyze the baryon mass from the viewpoint of its behavior as $N_c \to \infty$. In terms of its intrinsic quark content for the N_c quarks required for colorlessness, one would expect that the baryon mass $M \propto N_c$. In addition

one can look at a single-gluon exchange graph between two quarks within the baryon. The combined factors of $1/N_c$ from the coupling constants and $\frac{1}{2}N_c(N_c-1)$ from the number of possible quark pairs that can be selected to carry the one-gluon interaction again leads to $M \propto N_c$.

The same analysis applies to the exchange of quarks between two interacting baryons with a gluon transferred between them. Thus the baryon–baryon interaction in this analysis is also of order N_c. This suggests we shall have trouble ahead in considering the two-baryon force from a skyrmion point of view: we know from the start that the more subtle features of the NN force, such as its crucial medium-range attraction, are not of the order of the baryon mass ($\sim 1,000$ MeV) but rather some two orders of magnitude less (~ 20 MeV). The feature of NN interactions which *is* of order M is short-range repulsion, which indeed emerges naturally and easily with skyrmions.

To complete the discussion we must consider meson interactions, where a new feature enters, namely, the meson wave function in terms of contributions from all the available quark and antiquark pairs, N_c in number. This is

$$|\text{meson}\rangle = \frac{1}{\sqrt{N_c}}|\bar{l}l + \bar{m}m + \cdots + \bar{n}n\rangle \tag{44}$$

with N_c terms in the ket. Factors in the meson–nucleon coupling are then $1/N_c$ from the gluon–quark couplings, N_c from the choice of quark on which to paste the meson (quark–antiquark pair), N_c from the number of terms in the meson wave function, and $1/\sqrt{N_c}$ from the normalization of the wave function. Thus, meson–nucleon coupling goes as $\sqrt{N_c}$.

From considerations of the constituting quarks, one would expect the meson mass to be of order N_c^0 and this is borne out also by looking at one-gluon exchanges, where the two vertices yield $1/N_c$ and the two meson wave functions give $\left(N_c/\sqrt{N_c}\right)^2$, so that overall one has $\propto N_c^0$. Meson–meson interactions are

of order $1/N_c$, as is seen from the fact that once the initial meson quark–antiquark pairs are selected for each meson, the necessary constituents for the final mesons in the exchange graph are fixed. Thus there is a factor of $\sqrt{N_c}$ for each initial meson and two factors of $1/\sqrt{N_c}$ for the final mesons, as well as the usual $\left(1/\sqrt{N_c}\right)^2$ for the coupling of the transferred gluon: all in all a factor of $1/N_c$ for meson–meson scattering.

In summary, in the limit $N_c \to \infty$, QCD acts like a theory of weakly coupled mesons, which, if teated topologically, produces stable solitons that can be interpreted as baryons. Note that the meson-meson interaction

$$\propto 1/N_c \propto 1/f_\pi^2, \tag{45}$$

where f_π is the pion decay constant, so $f_\pi \propto \sqrt{N_c}$. In the space of mesons (or baryons), graphs involving gluons and quarks as explicit degrees of freedom are of higher order in N_c so meson degrees of freedom suffice suffice for the description and no colorless hadron connects to glue state.

4 Skyrmion generalities

4.1 The Skyrme lagrangian and the hedgehog ansatz

The lagrangian density originally proposed by Skyrme[1] (see also[7]) for the depiction of the nucleon through a solution in terms of meson fields with nontrivial topological properties is

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 = -\frac{F_\pi^2}{16}\mathrm{tr}(L_\mu L^\mu) + \frac{1}{32e^2}\mathrm{tr}[L_\mu, L_\nu]^2, \tag{46}$$

where F_π is the pion decay constant, with experimental value 186 MeV, and e is the Skyrme parameter which accompanies his stabilizing term with four derivatives. Here

$$L_\mu \equiv U^\dagger \partial_\mu U, \tag{47}$$

where $U(\mathbf{r},t)$ is the chiral field, at this stage taken in SU(2). For the $B=1$ static problem we take a time-independent hedgehog solution,

$$U(\mathbf{r},t) = \exp[i\boldsymbol{\tau}\cdot\hat{\mathbf{r}}F(r)], \tag{48}$$

where $F(r)$ is referred to as the profile function.

Let us explore briefly the features of the $B=1$ hedgehog solution in order to set the stage for general features of skyrmion solutions. Substituting eq. (48) into eq. (46) yields an expression for the lagrangian density in terms of the profile function,

$$\mathcal{L} = -\frac{F_\pi^2}{8}\left[\left(\frac{dF}{dr}\right)^2 + 2\frac{\sin^2 F}{r^2}\right] - \frac{1}{2e^2}\frac{\sin^2 F}{r^2}\left[\frac{\sin^2 F}{r^2} + 2\left(\frac{dF}{dr}\right)^2\right]; \tag{49}$$

this leads to a particle energy or mass

$$\begin{aligned} M = 4\pi\int_0^\infty r^2\,dr\Bigg\{ & \frac{F_\pi^2}{8}\left[\left(\frac{dF}{dr}\right)^2 + 2\frac{\sin^2 F}{r^2}\right] \\ & + \frac{1}{2e^2}\frac{\sin^2 F}{r^2}\left[\frac{\sin^2 F}{r^2} + 2\left(\frac{dF}{dr}\right)^2\right]\Bigg\}. \end{aligned} \tag{50}$$

One can now seek the extremum for the mass under variations of the profile function $F(r)$; this leads to the nonlinear differential equation

$$\begin{aligned} & \left(\frac{1}{4}F_\pi^2 r^2 + \frac{2}{e^2}\sin^2 F\right)F'' + \frac{1}{2}F_\pi^2\, rF' \\ & \quad + \frac{2}{e^2}\sin F\cos F F'^2 - \frac{1}{2}F_\pi^2 \sin F\cos F \\ & \quad - \frac{2}{e^2}\frac{\sin^3 F\cos F}{r^2} = 0. \end{aligned} \tag{51}$$

It is often convenient to cast this into a dimensionless form through the introduction of

$$\tilde{r} = eF_\pi r, \tag{52}$$

whence

$$\left(\frac{1}{4}\tilde{r}^2 + 2\sin^2 F\right)F'' + \frac{1}{2}\,\tilde{r}F' + \sin 2F F'^2 - \frac{1}{4}\sin 2F - \frac{\sin^2 F\,\sin 2F}{\tilde{r}^2} = 0. \tag{53}$$

Equation (52) underscores the fact that, so long as one works with the pure Skyrme lagrangian of eq. (46) without symmetry breaking or terms involving higher derivatives of U, all results involve dimensionless quantities which are scaled by eF_π for distances and by F_π/e for energies. Thus in a sense the success of the simplest version of the skyrmion as applied to the baryon lies in its ability to characterize the mass scale (~ 1000 MeV) and the size scale (~ 1 fm) with the one parameter e, assuming F_π is taken roughly at its experimental value. This also suggests a limitation of this version of the skyrmion, namely that at this level it does not allow for subtleties in baryon shapes beyond the mere determination of a root-mean-square radius.

Inspection of the lagrangian density of eq. (46) at the level of dimensions suggests that the first term $\mathcal{L}_2$ will ultimately yield a contribution to the energy that varies as the size of the baryon, while the second term $\mathcal{L}_4$ achieves its stabilizing effect because, having two additional derivatives, it varies as the reciprocal of the system size[d]. It is also clear from the expression for the baryon mass, eq. (50), that the term $F_\pi^2/4(\sin^2 F/r^2)r^2$ coming from $\mathcal{L}_2$ blows up as $r \to \infty$ unless

$$\lim_{r\to\infty} F(r) \to n\pi, \quad n = \text{integer}, \tag{54}$$

[d] In fact it is easy to show by a scaling argument that these two contributions must exactly equal each other for a proper solution of eq. (51) or (53). This is a handy check on numerics for the simplest case, and also hints[24] that a chiral expansion is inadequate if truncated at the level of four derivatives since one must expect the fourth-order contribution to be much less than the second-order one for that expansion to be valid.

while $((1/2e^2)(\sin^2 F/r^2)^2 r^2$ becomes infinite at $r = 0$ unless

$$F(0) = m\pi, \quad m = \text{integer}. \tag{55}$$

Further, since adding an integer multiple of π to $F(r)$ changes nothing in the differential equation for $F(r)$, only the difference between $F(0)$ and $F(r \to \infty)$, or $(m - n)$, can matter in fixing $F(r)$.

Indeed, the whole strength of the topological solution for baryons in terms of meson fields lies in the realization that there exists a topological constant determined by the structure of the solution for U rather than from the specifics of the dynamical solutions to the differential equation for it. The quantity in question is defined as

$$B^\mu = \frac{\epsilon^{\mu\alpha\beta\gamma}}{24\pi^2} \text{tr}\left[\left(U^\dagger \partial_\alpha U\right)\left(U^\dagger \partial_\beta U\right)\left(U^\dagger \partial_\gamma U\right)\right], \tag{56}$$

where $\epsilon^{\mu\alpha\beta\gamma}$ is the totally antisymmetric tensor density, and it is easily seen that

$$\partial_\mu B^\mu = 0. \tag{57}$$

Thus the current B^μ is conserved without regard to the equations of motion (i.e., it is not a Noether current.) One easily sees that the corresponding charge is conserved,

$$\frac{d}{dt}\int B^0(\mathbf{r}, t) d\mathbf{r} = -\int \boldsymbol{\nabla} \cdot \mathbf{B}(\mathbf{r}, t) d\mathbf{r} = \int_S \mathbf{B}(\mathbf{r}, t) \cdot d\mathbf{a} = 0, \tag{58}$$

using Gauss's theorem and the vanishing of U on the surface S at infinity. This absolute conservation of the charge density B^0 led Skyrme to identify its integral as the baryon number in this theory, so that B^μ is the baryon four-current. For the hedgehog case

$$\begin{aligned} B^0(r) &= -\frac{1}{2\pi^2 r^2} \sin^2 F \cdot F'(r) \\ &= -\frac{1}{4\pi^2 r^2} \frac{d}{dr}\left[F(r) - \frac{1}{2}\sin^2 F\right], \end{aligned} \tag{59}$$

so that the baryon number is given by

$$\int B^0 d\mathbf{r} = 4\pi \int_0^\infty B^0(r) r^2 dr = \frac{1}{\pi}[F(0) - F(\infty)] = m - n, \qquad (60)$$

which is simply the difference of the boundary values of the profile function divided by π. Thus for a single baryon the natural choice of boundary values is $F(0) = \pi$ and $F(r \to \infty) = 0$. In numerical solutions of eq. (53) one finds[7] simply a featureless fall-off of F from π to 0 as r is increased.

4.2 The mass spectrum and one-baryon observables

Thus far we have ignored time dependence and we also have not attempted to distinguish different baryon states through their different internal quantum numbers (spin and isospin in the present SU(2) case). Adkins, Nappi, and Witten[7] dealt with these issues by proposing a technique to rotate the hedgehog solution and project onto states of good baryon spin and isospin.[e] To do this, they introduce time dependence through a unitary transformation

$$U(\mathbf{r}, t) = A(t)\, U_0(\mathbf{r})\, A^\dagger(t), \qquad (61)$$

where $A(t)$ is a unitary SU(2) matrix. Were A fixed in time , the rotated U is a solution of the same differential equation as the unrotated one. With the introduction of time dependence we can use standard techniques to project out the quantum numbers we need, converting A into a dynamical variable and quantizing it (in the sense of first-quantization).

Substituting eq. (61) into the lagrangian density of (46) we get for the approximate lagrangian

$$L = \int \mathcal{L}\, d\mathbf{r} = -M + I \,\mathrm{tr}\left[\frac{dA}{dt}\frac{dA^\dagger}{dt}\right], \qquad (62)$$

[e] Criticism of this method is given in[25].

where M is the mass defined in eq. (50) and

$$I = \frac{2\pi}{3e^3 F_\pi} \int_0^\infty \sin^2 F \left\{ 1 + 4 \left[F'^2 + \frac{\sin^2 F}{\tilde{r}^2} \right] \right\} \tilde{r}^2 \, d\tilde{r}, \tag{63}$$

with the numerical result $I = 50.9\,(2\pi/3e^3 F_\pi)$. That there appear in eq. (62) only terms bilinear in the time derivatives of A is assured by Skyrme's original choice of $\mathcal{L}_4$.

The time-dependent transformation $A(t)$ is conveniently parametrized

$$A = a_0 + i\mathbf{a} \cdot \boldsymbol{\tau}, \tag{64}$$

where $a_0(t)$ and $a_i(t),\ i = 1, 2, 3$, now become the dynamical variables of the problem. The unitarity of A is assured by the side condition

$$a_0^2 + \mathbf{a}^2 = 1. \tag{65}$$

The lagrangian is

$$L = -M + 2I \sum_{i=0}^{3} \dot{a}_i^2, \tag{66}$$

whence the momenta conjugate to the a_i are

$$\pi_i \equiv \frac{\partial L}{\partial \dot{a}_i} = 4I\, \dot{a}_i. \tag{67}$$

The hamiltonian is given by

$$H = \sum_{i=0}^{3} \pi_i \, \dot{a}_i - L = 4I\dot{a}_i^2 - L = M + \frac{1}{8I} \sum_{i=0}^{3} \pi_i^2. \tag{68}$$

This is then first-quantized by taking

$$\pi_i \to -i \frac{\partial}{\partial a_i}, \tag{69}$$

whence

$$H = M + \frac{1}{8I} \sum_{i=0}^{3} \left(-\frac{\partial^2}{\partial a_i^2} \right). \tag{70}$$

By rotating the hedgehog form of eq. (48) in configuration space and in the SU(2) space one arrives at expressions[7] for the spin and isospin operators for the baryon,

$$J_k = \frac{i}{2}\left[a_k \frac{\partial}{\partial a_0} - a_0 \frac{\partial}{\partial a_k} - \epsilon_{klm}\, a_l \frac{\partial}{\partial a_m}\right] \tag{71}$$

and

$$I_k = \frac{i}{2}\left[a_0 \frac{\partial}{\partial a_k} - a_k \frac{\partial}{\partial a_0} - \epsilon_{klm}\, a_l \frac{\partial}{\partial a_m}\right]. \tag{72}$$

In turn one can use these to construct eigenfunctions of good spin and isospin. Since the hedgehog solution conserves the grand spin $\mathbf{K} = \mathbf{I}+\mathbf{J}$, all the relevant solutions will have equal spin and isospin, $j = t = 1/2,\ 3/2,\ 5/2,\ 7/2 \ldots$. Of these, only $j = t = 1/2$ and $3/2$ are physically relevant, the higher $5/2,\ 7/2, \ldots$ cases being ruled out either on the grounds that in the underlying QCD theory the three valence quarks cannot sum to spin or isospin higher than $3/2$, or because the states with $j = t = 5/2,\ 7/2, \ldots$ have widths so large that they should not be considered to be physical[26]. For the nucleon states, with $j = t = \frac{1}{2}$, one has

$$\begin{aligned} |p\uparrow\rangle &= \frac{1}{\pi}(a_1 + ia_2), \quad |p\downarrow\rangle = -\frac{i}{\pi}(a_0 - ia_3), \\ |n\uparrow\rangle &= \frac{i}{\pi}(a_0 + ia_3), \quad |p\downarrow\rangle = -\frac{1}{\pi}(a_1 - ia_2), \end{aligned} \tag{73}$$

and some sample cases for the Δ, with $j = t = \frac{3}{2}$, are

$$\begin{aligned} |\Delta^{++},\, s_z = \frac{3}{2}\rangle &= \frac{\sqrt{2}}{\pi}(a_1 + ia_2)^3, \\ |\Delta^{+},\, s_z = \frac{1}{2}\rangle &= -\frac{\sqrt{2}}{\pi}(a_1 + ia_2)\left[1 - 3\left(a_0^2 + a_3^2\right)\right]. \end{aligned} \tag{74}$$

The fact that these states are odd in a is what ensures here that they are fermionic in character[27]. The projection onto these states is made in the form of

expectation values with respect to them, the relevant integration being carried out over a unit sphere in four dimensions according to

$$\begin{aligned}\langle\cdots\rangle &= 2\int da_1\, da_2\, da_3\, da_0\ \delta\left(1-a_1^2-a_2^2-a_3^2-a_0^2\right)\cdots \\ &= \int da_1\, da_2\, da_3\ \frac{1}{|a_0|}\cdots\Bigg|_{a_0=\pm\sqrt{1-a_1^2-a_2^2-a_3^2}}. \end{aligned} \tag{75}$$

The energy spectrum (for the nucleon and delta states) is given by

$$E = M + \frac{1}{2I}j(j+1), \tag{76}$$

that is, the nucleon and delta masses are

$$M_N = M + \frac{3}{8I}, \qquad M_\Delta = M + \frac{15}{8I}. \tag{77}$$

One-baryon observables are extracted from the Noether currents generated by charge variations $\delta U = iQU$, where Q is the charge operator. The $V-A$ combination is given by

$$\begin{aligned} J_\mu^{V-A} &= \frac{iF_\pi^2}{8}\mathrm{tr}\left[(\partial_\mu U)\ U^\dagger Q\right] \\ &\quad - \frac{i}{8e^2}\mathrm{tr}\left\{\left[(\partial_\nu U)\ U^\dagger,\, Q\right]\left[(\partial_\mu)\ U^\dagger,\, (\partial_\nu)\ U^\dagger\right]\right\}, \end{aligned} \tag{78}$$

and the $V+A$ combination is arrived at by interchanging U and $U^\dagger$. More detailed expressions and numerical results for the root-mean-square isoscalar and isovector charge and magnetic radii, proton and neutron magnetic moments, axial coupling constant g_A, πNN coupling constant, $\pi N\Delta$ coupling constant, and $N\Delta$ transition magnetic moment are not shown here, having been extensively reviewed elsewhere[4,5,6,7]. They generally agree with experiment to within the expected $1/N_c \sim 33\%$ error.

4.3 Extensions of the lagrangian

It was early realized[28] that one could stabilize the skyrmion with terms representing ω-exchange in addition to or in place of the four-derivative form $\mathcal{L}_4$ of eq. (46) originally introduced by Skyrme. These are most easily taken in the limit of infinite ω mass as terms in the lagrangian density with six derivatives[29] in either of the two forms

$$\mathcal{L}_{6,1} \equiv -\epsilon_{6,1}^2 \mathrm{tr}\left[\mathcal{B}_\mu \mathcal{B}^\mu\right], \tag{79}$$

$$\mathcal{L}_{6,2} \equiv -\frac{\epsilon_{6,2}^2}{2} \mathrm{tr}\left[\mathcal{B}_\mu\right] \mathrm{tr}\left[\mathcal{B}^\mu\right], \tag{80}$$

where

$$\mathcal{B}^\mu \equiv \frac{\epsilon^{\mu\alpha\beta\gamma}}{24\pi^2}\left[\left(U^\dagger \partial_\alpha U\right)\left(U^\dagger \partial_\beta U\right)\left(U^\dagger \partial_\gamma U\right)\right] \tag{81}$$

differs from the similar definition of the baryon four-current of eq. (56) only in that the trace of eq. (56) is here suppressed so that it may be shown explicitly in the two different forms of eqs. (79) and (80). Two new parameters, $\epsilon_{6,1}$ and $\epsilon_{6,2}$ are here introduced. The additional index 1 or 2 on the quantities of eqs. (79) and (80) indicates whether one or two traces are eventually involved. The two forms of these equations are equivalent in SU(2), but not in SU(3), where, for example, the first contributes to nucleon spin content but the second does not.[30]

There are also alternative terms at the level of four derivatives, such as

$$\mathcal{L}_{4s} \equiv \frac{\gamma}{8e^2}\left[\mathrm{tr}\left(\partial^\mu U\, \partial_\mu U^\dagger\right)\right]^2, \tag{82}$$

where γ is again a new phenomenological parameter. This expression introduces terms with four time derivatives into the lagrangian, thus confusing the usual quantization procedures. It also tends to destabilize the skyrmion if its coefficient γ is taken large enough so as to overwhelm the other, repulsive

terms $\mathcal{L}_4$, $\mathcal{L}_{6,1}$, or $\mathcal{L}_{6,2}$. We thus entertain the expression of eq. (82) here reluctantly as perhaps having some limited phenomenological purpose. For the discussion of NN forces they are of some interest as possible sources of additional central attraction. In the context of single-nucleon uses of the skyrmion, they serve to stress the peculiar property of the skyrmion that it has a leg in each of two camps as it were: on the one hand, the Skyrme lagrangian is originally defined in terms of meson fields (more specifically, pion fields at the SU(2) level we are currently discussing), and on the other it applies to baryons through the topological aspect of the skyrmion solution. Thus one can extract[31] from $\pi\pi$ scattering data definite values for the parameters in the lagrangian $\mathcal{L}_2+\mathcal{L}_4+\mathcal{L}_{4s}$. Taking $F_\pi = 186$ MeV, one arrives at $\gamma = 0.16 \pm 0.04$ and $e = 5 \pm 2$, from which one can "predict" a nucleon mass $M_N = 880 \pm 300$ MeV, where the error noted is that arising only from uncertainties in the $\pi\pi$ data. Eventually, when we discuss the extensions of the skyrmion to SU(3) we shall also encounter an additional possible term[32] in the lagrangian with four derivatives which is equivalent to $\mathcal{L}_4$ in SU(2) but not in SU(3), in parallel to the situation of $\mathcal{L}_{6,1}$ and $\mathcal{L}_{6,2}$ noted above.

4.4 Ambiguities, parameter fitting, Casimir energy, etc.

This subsection is meant to make explicit certain ambiguities that are bound to accompany almost any discussion of applications of skyrmions to systems of baryons. We isolate this discussion here in order to give it appropriate emphasis, while saving the constant repetition of these caveats in the later text.

From the early uses of the skyrmion for single-baryon properties one finds, in broad outline, two somewhat different approaches to the determination of parameters: The first of these[7] uses the baryon mass spectrum to fix the co-

efficients of the various terms in the lagrangian. This usually leads to overly small values for the axial coupling constant g_A, for example, but this is partly to be expected in a theory that from the very beginning is prepared to tolerate inaccuracies of about 30 percent because of the assumption that $1/N_c = 1/3$ is a large number. The second approach, represented by ref.[33] say, attempts to fit various measurable baryon matrix elements—the so-called "observables"—letting the masses come out where they will. Since the agreement with the observables is bound to be partial at best, and since refined fitting procedures make little sense in a theory not expected to work to better than 30 percent, this method is also intrinsically limited.

The situation is made more complicated by uncertain features in the calculation of baryon masses. At the level of SU(3), with its richer baryon spectrum, it proves necessary to make a large subtraction ($\sim$ 500 to 1000 MeV) of a state with the quantum numbers of the vacuum,[34] so that in effect one is looking at mass splittings rather than at absolute positions. In a somewhat different approach,[35] the need for an even larger subtraction is justified in terms of ambiguities in operator ordering within the lagrangian that can express themselves as a large constant term in the energy. And of course there is the question of the skyrmion Casimir energy, which is estimated[24,36] to be quite large ($\sim$ 1000 MeV, again with large uncertainties).

Thus there seems to be no possible hard and fast rule for fixing skyrmion parameters. In practice one is more or less limited to using a mixture of criteria based on masses and observables. The extreme cases that are weighted strongly in one direction or the other can then be seen as a measure of the inaccuracy of the overall approach.

5 Skyrmions for the NN system with the product ansatz

In order to set sensible goals for the skyrmion as applied to two-nucleon systems, it is important to keep in mind the advantages and the intrinsic limitations of the skyrmion approach as well as the overall expectations for our understanding of the nature of the nucleon–nucleon force. As we have just stressed, the skyrmion is not expected to be reliable at a level much better than 30 percent, which certainly rules it out as a method for precision calculation of NN potentials for confrontation with experimental data. This restriction is made more acute by two further concerns: The skyrmion is basically directed at the energy scale of baryons, say 1000 MeV, whereas the NN force involves features that are typically one to two orders of magnitude smaller in energy and thus may well arise from differences between much larger energy quantities, resulting in a loss of precision in the calculation. In addition, the skyrmion is a low-energy theory; it involves an expansion in derivatives, or momenta, along the lines of that used in chiral perturbation theory. Thus the short-range features of the nucleon–nucleon interaction will surely not be well represented by it. From the beginning of the study of the application of the skyrmion to NN systems, we must look for a more or less qualitative understanding of the broad features of the NN force, and we must keep in mind that the method may ultimately fail at short ranges.

On the other hand, since the skyrmion provides a reasonable representation of πNN and $\pi N\Delta$ vertices while retaining strong roots in our fundamental understanding of strong interactions through QCD, it is in some ways an ideal tool for application to nuclear physics, where pion effects are often dominant[8,9] and quark features must clearly also be understood. In this sense it is the natural successor to the chiral bag model[7], which attempted to incorporate

these same elements in nuclear applications.

Thus in turning to applications of the skyrmion to the NN system we expect from it insights into the qualitative features of the NN force, and we would do well first to sketch the kinds of NN issues that should be addressed with it. For many years studies of the NN force have been based on a picture of meson exchanges,[38,39] which in more modern versions[40] also incorporates notions of chiral dynamics. The most solidly based aspect of meson exchange in the nucleon–nucleon force is of course that of one-pion exchange (OPE). As the lightest of the mesons, the pion dominates the long-range aspect of the NN potential and its role is easily verified in high-energy peripheral NN collisions. This is the feature that the skyrmion handles easily. Since the description of the pion as a Goldstone boson in QCD is complicated, it is advantageous to be able to isolate it before going on to other meson exchanges.

One-pion exchange is the major effect in deuteron binding but plays only a small role in heavier nuclei because its effects involve both spin and isospin variables which are quenched in nuclear systems with $B \geq 4$. There the crucial ingredients are the interplay between central attraction at intermediate NN separation distances and short-range repulsion. In early times, the former was thought to be provided by the exchange of a scalar σ meson, but when such a particle failed to turn up in experiment in the mass region (about 500 to 600 MeV) required to deal with intermediate ranges (~ 2 fm), a largely successful interpretation was developed in terms of two-pion exchange with intermediate states involving Δs. Short-range repulsion in the NN system was attributed to ω-meson exchange. This approach to the NN force based on two-pion exchange for attraction and omega-exchange for repulsion may well survive as supplying major ingredients for our understanding of that force, but it also seems likely that both pieces of the picture will be supplemented in an important degree

by quark dynamics; indeed it would be hard to understand how quark degrees of freedom could fail to enter in some measure at nuclear separation distances of 2 fm or less, where the individual nucleons are beginning to have sizable spatial overlap. In fact in the nonrelativistic quark model a significant source of NN central attraction at intermediate ranges has been found[41] in the form of mutual excitation of the two three-quark clusters that represent the baryons in the model. Such mechanisms must thus be entertained as possible supplements to two-pion exchange in producing central attraction. Similarly nonrelativistic quark models suggest[42] auxiliary sources of short-range repulsion stemming from the action of the color-magnetic force in QCD. This favors the occupation by quarks of the 2s states whose higher energy produces a repulsive effect at short ranges.

Against this background it is important to see what the skyrmion has to offer by way of descriptions of central attraction and short-range repulsion. The latter is in fact a very immediate consequence of the Skyrme model, which gives[33] very large energies for superposed hedgehog solutions with $B \geq 2$. The energies in question exceed by 1000 MeV or more the combined rest-mass energies of the baryons in these systems, and so are to be interpreted as large repulsion for small internucleon separations. Thus at the most fundamental level the main challenge for the skyrmion as applied to two-baryon systems is to offer a source of central attraction; this is the question that we shall address in the next sections. Once this has been dealt with, it is also of great interest to see what the skyrmion can offer by way of explanation of more subtle and more intrinsically dynamic issues, notably the origins of an attractive spin–orbit force which we shall address in section 9 below. Eventually one would hope to study the skyrmion also for whatever insights it may offer into nucleon behavior in larger nuclear systems (in the most immediate case, systems with

$B = 3$) and in nuclear matter, topics to which we turn in section 10. Since the present section covers older material which we ourselves have reviewed in the past,[18] our treatment of these topics is deliberately cursory, and is meant to be read in conjunction with that review; nonetheless we bring this material in brief form in order to have a relatively self-contained treatment and to set the stage for the discussion of more recent developments.

5.1 The product ansatz

From the very first stage of the introduction of his topological soliton, Skyrme was interested[1] in possible applications to the two-nucleon system. Towards this end he suggested the use of the product ansatz, in which the two-baryon system is represented by a topological field which is the product of two single-baryon fields,

$$U_{B=2}(\mathbf{r}, \mathbf{R}) = U_{B=1}(\mathbf{r} + \frac{1}{2}\mathbf{R})\, U_{B=1}(\mathbf{r} - \frac{1}{2}\mathbf{R}), \tag{83}$$

where $\mathbf{r}$ is the general skyrmion variable and $\mathbf{R}$ is the separation between the two baryon centers. This ansatz has the virtues that it automatically fulfills the correct baryon number condition $B = 2$ and has well-defined baryon centers by its very construction, in contrast to other approaches we shall encounter below. It also allows for simple physical interpretation when applied in the sense of a London–Heitler calculation in $B = 2$ systems.

It is crucial for the study of the NN system that we project the product ansatz onto nucleon states, if for no other reason that the dynamic admixture of Δs is, as noted above, often adduced as an important ingredient in providing central NN attraction, and thus we must be able to separate the nucleon from the Δ in addressing the NN force. This projection is accomplished by rotating

each member of the product ansatz separately according to

$$U_{B=2}(\mathbf{r},\mathbf{R}) = \left[A\,U_{B=1}(\mathbf{r}+\frac{1}{2}\mathbf{R})\,A^\dagger\right]\;\left[B\,U_{B=1}(\mathbf{r}-\frac{1}{2}\mathbf{R})\,B^\dagger\right], \qquad (84)$$

where A and B are the rotation matrices of eq. (61); it is sufficient to consider the relative rotation of the two baryons in order to project onto states with good nucleon quantum numbers, i.e., we may take

$$U_{B=2}(\mathbf{r},\mathbf{R}) = \left[U_{B=1}(\mathbf{r}+\frac{1}{2}\mathbf{R})\right]\;\left[C\,U_{B=1}(\mathbf{r}-\frac{1}{2}\mathbf{R})\,C^\dagger\right], \qquad (85)$$

where

$$C \equiv A^\dagger\,B = c_0 + i\boldsymbol{\tau}\cdot\mathbf{c}, \qquad (86)$$

with $c_0 = a_0 b_0 + \mathbf{a}\cdot\mathbf{b}$ and $\mathbf{c} = a_0\mathbf{b} - b_0\mathbf{a} + \mathbf{a}\times\mathbf{b}$.

The product ansatz may now be used[10,11] in a London–Heitler technique by choosing a separation distance $\mathbf{R}$ and calculating the resulting two-nucleon energy from the lagrangian, taking care to subtract the masses of the two nucleons (i.e., the energy at separation $\mathbf{R}\to\infty$). This requires the evaluation of integrals over the variable $\mathbf{r}$ and averaging over the rotation variables $\{c_0,\,\mathbf{c}\}$ in expectation values formed with the nucleon wave functions of eq. (73). Details are provided in the review of ref.[15]. At this level, and using the basic lagrangian of eq. (46), the skyrmion yields three components of the NN potential, namely,

$$V(\mathbf{R}) = V_C(R) + \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,[\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\,V_{SS}(R) + S_{12}\,V_T(R)]\,, \qquad (87)$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ are the Pauli spin and isospin operators and S_{12} is the usual tensor operator

$$S_{12} \equiv 3\,(\boldsymbol{\sigma}_1\cdot\mathbf{r})\,(\boldsymbol{\sigma}_2\cdot\mathbf{r}) - \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2. \qquad (88)$$

The three components of the force of eq. (87) are the central part $V_C(R)$, the spin–spin part $V_{SS}(R)$, and the tensor piece $V_T(R)$. These last two are dominated by OPE, and so should be well described by the skyrmion, as indeed

they prove to be[15] when compared with phenomenological spin–spin and tensor components of the NN force. The central potential is highly repulsive in its behavior for $R < 1$ fm, and no trace of attraction is found in that component for 1 fm $< R <$ 2 fm, where one knows it must occur from phenomenological potentials. In some sense it appears that the repulsion introduced through $\mathcal{L}_4$ in eq. (46) to stabilize the skyrmion spreads beyond the 1 fm range and overcomes any possible attraction. This occurs because the lagrangian is governed by two parameters which must account for baryon masses on the scale of 1000 MeV and baryon sizes on the scale of 1 fm. The smooth tail of the baryon profile then carries sizable repulsion out to the 2 fm range.

5.2 Higher-order derivatives in the lagrangian

Hints at possible ways to improve the skyrmion description of eq. (46) in order to identify a source of NN attraction are already found in the discussion of subsection 4.3, where we considered possible additional terms in the Skyrme lagrangian. For example, we noted there that the symmetric fourth-order term $\mathcal{L}_{4s}$ of eq. (82) is attractive—perhaps even too attractive in that for too large a parameter γ it can destabilize the skyrmion solution. One may thus incorporate the attractive $\mathcal{L}_{4s}$ along with the repulsive $\mathcal{L}_{6,1}$ of eq. (79) and attempt to play these off against each other in order to obtain attraction at medium NN separations while still keeping a stable skyrmion. This method is reviewed extensively in ref.[15]. It remains an important auxiliary ingredient in various other devices for producing attraction, but does not (ref.[43], also reviewed in[18]), in and of itself, yield attraction in $V_C(R)$, even after extensive search in the space of lagrangian parameters. Moreover, hopes that mutual distortions of the skyrmions (reviewed in[18]) might alleviate the situation come to naught since the dominant repulsion continues to prevail over the small

added attraction obtained form this source.

One could of course extend the skyrmion lagrangian to include the exchange of a wide variety of mesons, thus returning the problem—at least insofar as its NN applications are concerned—to the general structure of one-boson potentials[38,39]. This has been considered by the Orsay group (ref.[44], also reviewed in[15]) and, mostly for single-nucleon applications, by the Syracuse group (ref.[45], also reviewed briefly in[46]). The introduction of a large number of mesons in the skyrmion approach complicates the model somewhat, which leads to a partial loss of interpretive transparency and reduces the hope—an important motivation for studying the $B = 2$ system with skyrmions—of eventually applying the skyrmion in more complex nuclear situations.

In an approach that is related to the inclusion of a variety of exchanged mesons, one may consider[47,48,49] pion quantal fluctuations about the classical skyrmion solution. This should allow for the inclusion of two-pion exchange effects that have long been known[38] to be significant sources of central attraction, and indeed it does lead to components with attraction. However these do not readily overcome the repulsion that still persists at 2 fm within the product ansatz at least. Thus, while this mechanism is undoubtedly of continuing importance in the skyrmion context as well, it does not seem that it solves the overall problem of central attraction within that approach. These fluctuations also raise difficult formal and technical problems, in that they must be carefully chosen so as to be orthogonal to the variations of the skyrmion that lead to the classical equations of motion.

It has been noted by several authors that one should make at least a minimal, algebraic correction for working with $N_c \to \infty$ in the skyrmion approach[10,64,65,66,67]. Moreover, the correction in question acts to increase somewhat the central NN attraction found with the skyrmion. As we have

seen in subsection 4.2, rotating the skyrmion leads to states with $j = t = 5/2,\ 7/2 \ldots$ which are not physically realized, an effect which it is natural to assume is an artifact of taking $N_c \to \infty$ and thus exceeding the natural spin-isospin structure for the three quarks involved when $N_c = 3$, which would cut off at $j = t = 3/2$. Amado and his collaborators[65,66,67] (reviewed in[21]) pointed out that a natural way of handling this cutoff could be achieved by examining the dynamical group symmetry of skyrmion rotation: The rotational algebra of $\mathrm{SU}(2)_j \times \mathrm{SU}(2)_t$ is embedded in U(4) and the symmetric (N_c) representation of this group is then taken as the lowest energy band. The nucleon belongs to this band, as do its rotational excitations with $j = t = 1/2,\ 3/2 \ldots, (N_c - 2)/2$. The rotational spectrum arising from this implementation agrees with the quark model prediction of maximal spin and isospin $j = t = N_c/2$. This allows the determination of correction factors for various spin-isospin operators and thus for the corresponding single-baryon observables as well as for the $B = 2$ potentials, yielding increased central attraction for the NN system.

Last among the refinements within the general scope of the product ansatz we note studies of the dynamic features of the skyrmion mass-tensor parameter, which may be obtained by "dragging" the hedgehog[68,69,70,71,72,73] (reviewed briefly in[21]). In the context of the $B = 2$ problem, these contribute velocity-dependent terms in the NN force some of which prove to be attractive but not very large. The complete analysis of translational and rotational collective motion for two skyrmions, even within the confines of the product ansatz, proves to be a problem of some intricacy. This is evident even in the relatively simple context of the spin–orbit force (section 9) where these ingredients are necessary in order to identify that component of the force. There have also been efforts made[74] to study the NN force in the context of alternative stabilization mechanisms[75] for the skyrmion that may greatly simplify the calculational

tedium of such studies.

5.3 Baryon resonance admixtures

It has long been accepted wisdom[38] that the attraction in the central NN potential represented by so-called σ-exchange in fact arises because of two-pion exchanges which can simulate the σ effects. It is essential that in the application of box graphs of two-pion exchange to the construction of the NN potential at least one of the intermediate baryon lines refer to excited states and not to the nucleon itself. This is because in the use of the Schrödinger equation to solve the two-nucleon problem one includes all iterations of graphs with two-nucleon lines, so that the construction of the potential itself must omit these in order to avoid double counting. The physical picture is then that at intermediate ranges the two-nucleon system is partly an $N\Delta$ or $\Delta\Delta$ system or indeed some other $B = 2$ system containing higher excitations of the nucleon. This suggests that in using the skyrmion for the derivation of the NN potential it is in order to consider the admixture of $N\Delta$ and $\Delta\Delta$ states[43] into the wave function,

$$|\Psi(R)\rangle = \alpha(R)|NN(R)\rangle + \beta(R)|N\Delta(R)\rangle + \gamma(R)|\Delta\Delta(R)\rangle. \qquad (89)$$

Here we once again have in mind the use of the London–Heitler technique. That is to say, the calculation[43] proceeds by fixing the internucleon separation distance R and then minimizing the system energy with respect to changes in the admixture coefficients $\alpha(R)$, $\beta(R)$, and $\gamma(R)$ at the value of R in question. For this mechanism to yield appreciable intermediate-range attraction, it is necessary[50] to treat carefully the action of the tensor force in admixing spin states. Even then only very weak attraction ($\sim$ 2 to 3 MeV)—and that only for extreme skyrmion parameters which tend to overaccentuate the role of the

Δ—is achieved[50,51].

The picture in this regard is improved considerably if one entertains the admixture of the Roper $N(1440)$ resonance by extending eq. (89) to include these. The initial motivation to consider this mechanism for central attraction in the NN force arose[52] from the realization that a similar device appears importantly in deriving central attraction in the nonrelativistic quark model, where it was found[41] that the internal excitation of the individual three-quark clusters making up each nucleon had to be allowed in order to obtain a reasonable result for $V_C(R)$. The description of the Roper excitation as a vibrational state was given early on by a number of authors[53,54,55,56,57,58,59,60,61]. We follow here the method and notation of ref.[58] (see also the review in[18]).The hedgehog ansatz for the individual baryon is written as

$$U_\lambda(\mathbf{r}) = U_0(\mathbf{r}\mathrm{e}^{\lambda(t)}), \tag{90}$$

where the time-dependent scaling parameter $\mathrm{e}^{\lambda(t)}$ allows for a breathing-mode vibration of the hedgehog. This new ansatz is then substituted into the skyrmion lagrangian and the time-dependent terms are analyzed in a harmonic approximation for λ together with the rotational prescription of eq. (61). The hamiltonian with rotational and vibrational degrees of freedom is then of the form

$$H_\lambda = \Sigma(\lambda)\,\dot{\lambda}^2 + V(\lambda), \tag{91}$$

where

$$\Sigma(\lambda) = f_1(\lambda) + \frac{j(j+1)}{4I^2(\lambda)} f_3(\lambda) \tag{92}$$

and

$$V(\lambda) = M(\lambda) + \frac{j(j+1)}{2I(\lambda)}, \tag{93}$$

with j the baryon angular momentum as in eq. (76), and $M(\lambda)$ and $I(\lambda)$ are the skyrmion mass and moment of inertia. These last were presented in eqs.

(50) and (63) for the case of the basic skyrmion lagrangian of eq. (46), which here may be modified to include the effects of higher-derivative terms (section 5.2). Expressions for these more complete forms as well as for $f_1(\lambda)$ and $f_3(\lambda)$ are given in refs.[18,52].

One then proceeds[58] by quantizing about the classical minimum

$$\left.\frac{dV}{d\lambda}\right|_{\lambda=\lambda_0} = 0, \tag{94}$$

which defines λ_0, and in the harmonic approximation arrives at

$$H(\lambda) = \Sigma(\lambda_0)\,\dot{\lambda}^2 + V(\lambda_0) + \frac{1}{2}\left.\frac{d^2V}{d\lambda^2}\right|_{\lambda=\lambda_0}(\lambda-\lambda_0)^2 + \cdots. \tag{95}$$

The usual first quantization in λ leads to the oscillator spectrum

$$E = V(\lambda_0) + (n+\frac{1}{2})\Omega, \quad n = \text{integer}, \tag{96}$$

with oscillator frequency

$$\Omega = \left[\frac{k}{2\Sigma(\lambda_0)}\right]^{1/2}, \quad k \equiv \left.\frac{d^2V}{d\lambda^2}\right|_{\lambda=\lambda_0}. \tag{97}$$

Eigenvalues with $n = 0$ correspond to the nucleon and Δ—the latter being distinguished, of course, by its rotational motion—and those with $n = 1$ correspond to the Roper $N(1440)$ and the Δ-like particle built upon it, $\Delta(1670)$.

With reasonable parameters, the extension of eq. (89) to include the admixture of these two states then yields about 5 MeV of attraction in $V_C(R)$, which is a not unreasonable well depth given the fits of modern NN potentials such as that of Paris[62] or of Argonne[63] with well depths of 5 to 20 MeV. There is also a certain satisfaction in the fact that the skyrmion and the nonrelativistic quark model find that the same basic feature yields attraction, namely, the internal excitation of each baryon participating in the interaction between the two. This qualitative feature drives home the intrinsic subtlety of the origins

of central attraction in two-baryon systems—a feature crucial to the binding of nuclei, for instance. It also hints at a finding of the exact skyrmion calculations (section 7): there too the departure of the skyrmion at short internucleon separations from its structure at $R \to \infty$ is what yields attraction, so that at least at a qualitative level many of the different approaches may in fact be convergent.

6 The dilaton

6.1 Dilaton generalities

We shall open our considerations on dilatons by recalling briefly the ingredients of scale invariance in the case of a classical, massless field. We use for this purpose the scalar fields we discussed in the general context of solitary waves, but ultimately we shall have reference to QCD. Thus we take a general lagrangian of the form

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi(x)\, \partial^\mu \phi(x) - V[\phi(x)], \tag{98}$$

where we shall consider more specifically the free lagrangian for a particle with mass,

$$\mathcal{L}_0 = \frac{1}{2}\partial_\mu \phi\, \partial^\mu \phi - \frac{1}{2}m^2\phi^2, \tag{99}$$

and a massless ϕ^4 theory,

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi\, \partial^\mu \phi - \frac{\lambda}{4!}\phi^4. \tag{100}$$

Recall that the action—dimensionless—is $S \equiv \int \mathcal{L}\, d^4x$, so that it is easy to read off the dimensions of the f field, namely, $[\phi] = \text{energy} = 1/\text{length}$.

An infinitesimal dilatation is given by

$$x'^\mu = (1+\alpha)x^\mu, \qquad \phi' = (1-\alpha)\phi, \tag{101}$$

and leaves the action invariant for $d = 4$. The Noether current corresponding to this symmetry is

$$j^\mu = -\left[g^\mu_\rho\,\mathcal{L} - \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\partial_\rho\phi\right]\frac{\delta x^\rho}{\delta\omega} - \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\frac{\delta\phi}{\delta\omega}, \tag{102}$$

or, suppressing α,

$$\begin{aligned} j^\mu_{\rm D} &= \left[-g^\mu_\rho\,\mathcal{L} + \partial^\mu\phi\,\partial_\rho\phi\right]x^\rho - \left(\partial^\mu\phi\right)\left(-\phi\right) \\ &= j^\mu_\rho\,x^\rho + \frac{1}{2}\partial^\mu\phi^2, \end{aligned} \tag{103}$$

where j^μ_ρ is the energy-momentum tensor (i.e., the Noether current for $x'^\mu = x^\mu + \epsilon^\mu$ and $\phi' = \phi$). Thus

$$\partial_\mu j^\mu_{\rm D} = j^\mu_\mu + \frac{1}{2}\partial^\mu\partial_\mu\phi^2 \tag{104}$$

since

$$\partial_\mu j^\mu_\rho = 0. \tag{105}$$

For

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{\lambda}{4!}\phi^4 \tag{106}$$

we then have

$$\begin{aligned} j^\mu_\mu &= -g^\mu_\mu\mathcal{L} + \partial^\mu\phi\,\partial_\mu\phi \\ &= -4\left(\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{\lambda}{4!}\phi^4\right) + \partial^\mu\phi\,\partial_\mu\phi \\ &= -\partial_\mu\phi\,\partial^\mu\phi + \frac{\lambda}{6}\phi^4. \end{aligned} \tag{107}$$

The equation of motion is

$$\partial_\mu\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} - \frac{\partial\mathcal{L}}{\partial\phi} = \partial_\mu\partial^\mu\phi + \frac{\lambda}{6}\phi^3 = 0, \tag{108}$$

so that

$$\begin{aligned}\partial_\mu j^\mu_{\rm D} &= j^\mu_\mu + \frac{1}{2}\partial_\mu\partial^\mu\phi^2 \\ &= j^\mu_\mu + \phi\partial^\mu\partial_\mu\phi + \partial_\mu\phi\,\partial^\mu\phi \\ &= \phi\,\partial^\mu\partial_\mu\phi + \frac{\lambda}{6}\phi^4 = 0,\end{aligned} \tag{109}$$

and the dilatation current is indeed conserved.

If a mass term is present,

$$\partial_\mu j^\mu_{\rm D} = m^2\phi^2 \neq 0 \tag{110}$$

because a dimensioned parameter appears in $\mathcal{L}$ and the corresponding action behaves as

$$\begin{aligned}S_{\rm m} &= -\frac{1}{2}m^2\int\phi^2 d^4x \\ &\to (1+\alpha)^2\Big(-\frac{1}{2}m^2\int\phi^2 d^4x\Big).\end{aligned} \tag{111}$$

For higher spin, as in the QCD case, dilatation is linked to a traceless energy-momentum tensor.

The crucial consideration for our purposes is that when quantal effects are introduced dilatation invariance breaks down even if it was present at the classical level. This is easily seen within the same ϕ^4 theory by merely considering the lowest, one-loop corrections, which have the effect of replacing the lowest-order ϕ^4 coupling constant λ_0 with

$$\lambda = \lambda_0 - \frac{i}{2}\lambda_0^2\big[I(p_1+p_2) + I(p_1+p_3) + I(p_2+p_3)\big], \tag{112}$$

where

$$\begin{aligned}I(p) &= \int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2+i\epsilon)[(k+p)^2+i\epsilon]} \\ &= \frac{i}{16\pi^2}\log\frac{\Lambda^2}{-p^2+i\epsilon}.\end{aligned} \tag{113}$$

Thus

$$\lambda = \lambda_0 - \frac{3\lambda_0^2}{32\pi^2}\log\frac{\Lambda^2}{\mu^2}, \quad p_1^2 = p_2^2 = p_3^2 = p_4^2 = -\mu^2, \tag{114}$$

so that the need for a cutoff Λ introduces a scale which was hidden in the classical theory. Renormalization can lead to spontaneous symmetry breaking and a massless classical theory may acquire a mass in the quantum theory. Dilatation invariance and tracelessness are then broken. This happens in QCD: that is why hadrons, which are made up of (nearly) massless quarks and thus should be expected to be (nearly) massless themselves, have definite, nonzero masses.

6.2 The dilaton in the context of skyrmions

As we have hinted, (see, e.g.,[77,78,79] for further detail) QCD with massless quarks is scale invariant when treated as a classical theory and loses this property upon quantization. Accompanying this is the trace anomaly, whereby the trace of the energy-momentum tensor $T^{\mu\nu}$, which vanishes classically, is given after quantization by

$$T^\mu_\mu = \partial_\mu D^\mu = -\frac{9\alpha_s}{8\pi} G^a_{\mu\nu} G^{a\,\mu\nu} \equiv \psi^4, \tag{115}$$

where $D^\mu (= T^{\mu\nu} x_\nu)$ is the dilatation current, α_s is the QCD coupling constant, $G^a_{\mu\nu}$ is the gluon field, and ψ is an order-parameter field—the dilaton—which represents the scalar glueball formed from the contraction of the two gluon fields; ψ is taken in the fourth power in anticipation of its use as a scalar field of dimension 1. It was early pointed out that an effective lagrangian based on this scalar field can be used to incorporate the trace anomaly[80], and the consequences of such an approach for scalar glueballs were studied some time thereafter[81], along with modifications to the Skyrme lagrangian in order to incorporate there first the appropriate scale invariance and then the trace

anomaly[82,83]. The physical picture for the skyrmion plus dilaton is that a kind of bag is formed by the dilaton field such that the skyrmion is located within a region where the gluon condensate is suppressed. In many ways this picture is the natural extension of previous studies based on chiral or cloudy bags[37]. It also answers for one of the obvious limitations of the skyrmion as it stands alone, namely, that there is only one length parameter in the basic model, so that there is no well-defined concept of a "surface thickness" or baryon edge. Since the skyrmion is characterized by the lagrangian of eq. (46) in terms of the energy unit F_π/e and length unit eF_π, as noted below eq. (53), the need to fit single-baryon properties automatically introduces a scale of hundreds of MeV for the energy and about a fermi for sizes. Thus when we search for attraction in the NN force we must overcome the tail of the very strong repulsion at zero separation which still persists strongly in the range of 2 fm or so of relevance for attraction. Obviously, the sharpening of the baryon surface will help to reduce this problem.

Schechter[80] and later his coworkers[81,82,83] introduced a lagrangian for the dilaton field through the consideration of the trace anomaly. If terms with no more than two time derivatives appear, the most general form allowed is

$$\mathcal{L}_{\rm dilaton} = \mathcal{L}_{2\ \rm dilaton} - V_{\rm dilaton}(\psi) = \frac{1}{2}b^2\partial_\mu\psi\partial^\mu\psi - \frac{1}{4}\psi^4\log\left[\frac{\psi^4}{\Lambda^4}\right], \qquad (116)$$

where b is a dimensionless constant and Λ is a constant with dimensions of energy. For the mean-field case, eq. (116) then leads to $\langle\psi\rangle = \Lambda/\mathrm{e}^{1/4}$.

Given the dilaton lagrangian, it is easy to modify[82,83] the basic Skyrme lagrangian (46) in order to incorporate the trace anomaly, namely,

$$\mathcal{L} = \mathcal{L}_{\rm dilaton} - \frac{F_\pi^2}{16}\frac{\psi^2}{\langle\psi\rangle^2}\mathrm{tr}(L_\mu L^\mu) + \frac{1}{32e^2}\mathrm{tr}[L_\mu, L_\nu]^2, \qquad (117)$$

where the kinetic term is modified on scaling grounds by a term with ψ^2 and the Skyrme stabilizing term requires no such change. It is common practice to

replace ψ with an exponentiated scalar field σ,

$$\psi \equiv \langle\psi\rangle \, \mathrm{e}^{\sigma}, \tag{118}$$

whence the dilaton potential is

$$V(\sigma) = \frac{1}{4} \, C_G \, \mathrm{e}^{4\sigma} (1 - 4\sigma), \tag{119}$$

taking the pion mass to vanish. The overall lagrangian, including the terms with six derivatives of section 5.2, is then

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{2 \text{ dilaton}} + \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 - V(\sigma) \\ &= \mathrm{e}^{2\sigma} \left[\frac{1}{2} \Gamma_0^2 \, \partial_\mu \sigma \, \partial^\mu \sigma - \frac{F_\pi^2}{16} \mathrm{tr}(L_\mu L^\mu) \right] + \frac{1}{32 e^2} \mathrm{tr}[L_\mu, \, L_\nu]^2 \\ &- \epsilon^2 \mathrm{e}^{-2\sigma} B_\mu B^\mu - \frac{C_G}{4} [1 + \mathrm{e}^{4\sigma} (4\sigma - 1)]. \end{aligned} \tag{120}$$

The two parameters Γ_0 and C_G are related to the glueball—or scalar field—mass through $m_\sigma^2 = 4 C_G / \Gamma_0^2$. For frequently-used values for these parameters this may yield a rather low mass for the σ glueball field, which one must assume then mixes strongly with $\pi\pi$-exchange simulations of σ-like effects; nonetheless, the virtue of the dilaton approach is that is provides a natural source for this scalar rather than lying on a completely ad hoc introduction of it into the theory. (Other criticisms of the use of the dilaton in nuclear-matter applications have been noted in ref.[84].)

Once the dilaton is coupled to the skyrmion, it provides a very natural source of central attraction[85]. The usual product ansatz must be modified by an assumption of dilaton additivity,

$$\sigma_{B=2} = \sigma_1 + \sigma_2, \tag{121}$$

which is assumed to be of sufficient accuracy for purposes of calculating the NN force since mixed terms are cut out in the function $\psi_{B=2} = \langle\psi_{B=2}\rangle \, \mathrm{e}^{\sigma_{B=2}}$.

Indeed, ref.[85] finds some 35 MeV of attraction in the central NN potential for $r \sim 1.8$ fm. We return to the application of the dilaton to the NN potential in section 9 below, both with regard to its use for $V_C(R)$ and for the spin–orbit potential.

7 The exact skyrmion solution for $B = 2$

Recently solutions of the full $B = 2$ skyrmion problem in three dimensions (or, exploiting the cylindrical symmetry of the problem, in two dimensions) have become increasingly available. The review of Walhout and Wambach[22] devotes itself in considerable degree to this topic, in which these authors have themselves been very much involved. Therefore the first part of the present section is to be read in conjunction with that review; more recent developments stemming from Manton and coworkers will also be touched upon briefly below.

The earliest solutions[33] for the $B = 2$ skyrmion, noted briefly in section 5 above, dealt with a spherically symmetric profile function which automatically pertains to two baryons centered on the same point. Apart from the fact that such a configuration has very large energy ($\sim 3M_N$), related to the strong short-range repulsion of the two-nucleon system, this configuration is of little physical interest; it is, of course, highly unstable. Early exact numerical solutions[86,87,88,89] for the $B = 2$ system were obtained by relaxing the full static equations of motion on a grid within a particular geometry. These solutions have a toroidal shape, the hole in the middle being, presumably, the reflection of short-range repulsion. They do not show any residual signs of the presence of two separate nucleons, which raises the criticism noted by Manton[76] that this solution does not allow sufficient degrees of freedom to describe correctly the complete dissolution into two baryons.

Subsequent exact numerical studies were usually based on an assumption

of adiabaticity, whereby one introduces constraints to fix the collective coordinates and solves the static equations of motion (as opposed to trying to follow the full time evolution of two skyrmions aimed at each other; see[86,90,91,92]). The fixing of coordinates pertains both to the relative orientation of the skyrmions and to the distance between their centers. The latter may be determined, say, through Lagrange multipliers,[f] while the relative orientations between the two skyrmions are chosen in the same manner as for the instanton solution. The solutions[g][22,99] then show two separated nucleons which coalesce at about 1.2 fm for the maximally attractive relative orientation of the skyrmions, and then meld into a toroid below that "separation" distance. A very small amount of central attraction may be found in this approach; this is somewhat increased by admixing[101,102,103] $N\Delta$, ΔN, and $\Delta\Delta$ states, as discussed in subsection 5.3. Attraction is also augmented by incorporating[100] the dilaton, in which case the "bags" provided by the dilaton merge at about 1.4 fm, and an attractive central component on the scale of 20 MeV is found. This is further enhanced by again considering the admixture of states containing Δs.

The ability to carry out these difficult calculations is encouraging, and no doubt goes far towards providing a mechanism of central attraction in two-nucleon interactions in the skyrmion approach, but more must be done to refine them in order to be sure that all artifacts of the numerical procedures have been fully eliminated.[h] This may also lead to a reliable assessment as to which ingredients are essential to obtain central attraction. The connections with others approaches should also be pursued further in the hope of eventually

[f] A hybrid model[93,94] involving the full numerical solution of a product ansatz form has the advantage that it automatically places the skyrmions on well-defined centers and maintains $B = 2$ automatically.

[g] Related approximative calculations involving simpler numerical methods have also been carried out[95,96,97,98].

[h] In particular, it is even difficult[104] fully to ascertain if the two-skyrmion solution remains on the same manifold as the free skyrmions approach each other and merge into a torus.

constructing a model simple enough for application in more complex nuclear situations.

Quite recently Leese, Manton, and Schroers[105] have produced a skyrmion solution for the deuteron carried out on a ten-dimensional manifold which yields properties close to those of the physical deuteron.[i] It should be emphasized that this is the first case of a realistic representation of the deuteron within the skyrmion approach. The NN system should ideally be calculated on a twelve-dimensional manifold in order to include full freedom for the locations of the two skyrmions and for their SU(2) orientations. This calculation still proves prohibitively difficult, however, and so ref.[105] uses the lowest-dimensional manifold that still includes both a toroidal configuration of minimal energy and configurations that approximate the infinite separation of two skyrmions oriented in the maximally attractive relative direction in SU(2). To perform the calculation, an instanton assumption is made and the maximally attractive orientation—rotation by π about the axis perpendicular to the line joining the skyrmions—is taken. The geometry that underlies the instanton is somewhat complicated in order to accommodate the limiting toroidal and separated configurations. The technical aspect of the calculation then involves evaluating the lagrangian after restricting to that geometry, and generating the hamiltonian with first-quantized collective degrees of freedom. Numerical values for deuteron properties in this approach and in that of ref.[89] are shown in Table 1, from which it is seen that results from the ten-dimensional manifold M_{10} are far closer to those of experiment than are the ones arising[89] from an eight-dimensional one M_8, because the larger dimensionality is necessary in order to allow the skyrmions to sample a space that allows also for genuine separation of the two nucleon centers.

[i] A discussion of this approach, along with criticism of it, has been given recently in[106].

Property	M_{10} (ref.[105])	M_8 (ref.[89])	Experiment
ϵ_D [MeV]	-6.18	-158	-2.225
r_c [fm]	2.18	0.92	2.095
Q [fm^2]	0.83	0.082	0.2859
μ [n.m.]	0.55	0.74	0.8574

Table 1: Deuteron properties. Here M_{10} refers to a ten-dimensional manifold and M_8 to an eight-dimensional one; ϵ_D is the deuteron binding energy, r_c is the charge radius, Q is the quadrupole moment, and μ the magnetic moment. From ref. [105].

8 Systems with nonzero strangeness

8.1 General issues

If nothing else, this review should by now have convinced the reader that the origins of central attraction at medium ranges in the NN force are still unclear. We have a loose understanding of how it might come about in models that base themselves on QCD, but little more than that. And the skyrmion, with perhaps the best claim amongst such models to a genuine QCD pedigree, mostly serves to stress that central attraction is a subtle affair, which may derive from some or all of the proposed mechanisms we have noted, or, indeed, from others. Nor is it clear if the problems lie in the basic notion of the skyrmion (i.e., of the large N_c limit leading to mesonic degrees of freedom interpreted as baryons through a topological argument) or in the particular lagrangian proposed for it (i.e., might medium ranges require more field derivatives?). All of this makes

it especially interesting to consider systems with nonzero strangeness, where very similar mechanisms may enter and one may be able to highlight specific features of the $B = 2$ interaction by exploiting the different quantum numbers of the $S \neq 0$ systems.

8.2 SU(3) extensions of the skyrmion

Early efforts to extend the skyrmion to SU(3) failed because of various technical errors in treating flavor symmetry breaking. With the work of Yabu and Ando[34] and of Callan, Hornbostel, and Klebanov[107] the proper treatment of this crucial feature, within various possible theoretical approaches to the problem, was given, and a good descriptions of the single-baryon system were obtained. We shall here follow the method of Yabu and Ando, which is somewhat more general. In addition to the $\mathcal{L}_2$ and $\mathcal{L}_4$ terms of eq. (46), these authors include[j,34] the symmetry-breaking term

$$L_{\rm SB} = \int d\mathbf{r}\ F_\pi^2 \left\{ \frac{1}{32}\left(m_\pi^2 + m_\eta^2\right) \mathrm{tr}(U + U^\dagger - 2) + \frac{\sqrt{3}}{24}\left(m_\pi^2 - m_K^2\right) \mathrm{tr}\left(\lambda_8 (U + U^\dagger)\right) \right\}, \tag{122}$$

where here and in the following the λ_i, $i = 1, \ldots, 8$ refer to the eight Gell-Mann SU(3) matrices. The SU(3) flavor symmetry is broken here by the appearance of different masses—m_π, m_η, and m_K for the π, η, and K mesons—of the octet. We note that further symmetry breaking could be included by taking also different decay constant for the various mesons, rather than using F_π for

[j] It was early known[108] in the context of chiral perturbation theory, but realized[32,109] only rather late for skyrmions, that a stabilizing form additional to that of eq. (46) can occur in SU(3), namely, $\mathcal{L}_{4b} = \mathrm{tr}\,(L_\mu\, L_\nu)\,\mathrm{tr}\,(L^\mu\, L^\nu) - \mathrm{tr}\,(L_\mu\, L^\mu)\,\mathrm{tr}\,(L_\nu\, L^\nu)$, which is equivalent to $\mathcal{L}_4$ in SU(2). Unfortunately, this new term offers no advantages where the search for central NN attraction is concerned[110]. (Recall that we encountered the existence of terms equivalent in SU(2) but not in SU(3) also for cases with six derivatives in subsection 4.3.)

all of them. The symmetry-breaking term is taken such that when the field is expanded as $U = \exp\left[2i/F_\pi \sum_{i=1}^{8} \lambda_i \phi_i\right]$ about the classical vacuum $U = 1$ one can immediately read off the meson masses; the form of eq. (122) then automatically satisfies the Gell-Mann–Okubo mass formula

$$m_\pi^2 + 3m_\eta^2 - 4m_K^2 = 0. \tag{123}$$

For the SU(3) case it is also necessary to include the Wess–Zumino term (which vanishes in SU(2)),

$$\begin{aligned} L_{\rm WZ} = \frac{N_c i}{240\pi^2} \int_Q d\Sigma^{\alpha\beta\gamma\delta\epsilon} \\ \times \mathrm{tr}\left[\left(\partial_\alpha U\right)U^\dagger \left(\partial_\beta U\right)U^\dagger \left(\partial_\gamma U\right)U^\dagger \left(\partial_\delta U\right)U^\dagger \left(\partial_\epsilon U\right)U^\dagger\right], \end{aligned} \tag{124}$$

which is given as an integral over a five-dimensional disk Q, the boundary of which is the compactified space-time four-sphere.

The embedding of the SU(2) hedgehog into SU(3) is done through

$$U_0(\mathbf{r}) = \exp\left[i \sum_{i=1}^{3} \lambda_i \hat{\mathbf{r}}_a F(r)\right]. \tag{125}$$

The introduction of collective rotational coordinates goes through as for SU(2), with, of course, the extra degrees of freedom arising in SU(3). This leads to the hamiltonian

$$\begin{aligned} H = M_{\rm cl} + \frac{1}{2}\left[\frac{1}{\alpha^2} - \frac{1}{\beta^2}\right] C(\mathrm{SU_R}(2)) - \frac{3}{8\beta^2} + \frac{1}{2\beta^2} C(\mathrm{SU_L}(3)) \\ + \frac{1}{2}\gamma\left(1 - D^8(A)\right), \end{aligned} \tag{126}$$

where $C(\mathrm{SU_R}(2))$ and $C(\mathrm{SU_L}(3))$ are the Casimir operators for $\mathrm{SU_R}(2)$ and $\mathrm{SU_L}(3)$, and $D^8(A) = \frac{1}{2}\mathrm{tr}\left(\lambda_8 A^\dagger \lambda_8 A\right)$, with A the rotation matrix as in eq. (61) but now in SU(3). The moment of inertia for SU(2) repeats that of eq. (63), $\alpha^2 = \lambda$, while that for SU(3) is given by

$$\beta^2 = \frac{\pi}{2e^3 F_\pi} \int_0^\infty (1 - \cos F)\left[\tilde{r}^2 + \left(2\sin^2 F + (\tilde{r}F')^2\right)\right] d\tilde{r}, \tag{127}$$

and the mass term is

$$\gamma = \frac{4\pi}{3e^3 F_\pi}\left(m_K^2 - m_\pi^2\right)\int_0^\infty \tilde{r}^2(1-\cos F)d\tilde{r}. \tag{128}$$

Energy splittings for the baryon octet and decuplet spectra found[34] with this method agree remarkably well with experiment, although there is a persistent need to adjust the kaon mass upwards for this purpose and to subtract the energy of a state having the quantum numbers of the vacuum, this subtraction typically being on the scale of half the nucleon mass or so. As with the SU(2) case, fitting to the mass spectrum yields too low a value for F_π, although this situation can be improved somewhat using an approach due to Kanazawa[35], which yields[111,30] reasonable results for nucleon observables.

In order to give some impression of the degree to which the skyrmion can match SU(3) expectations we show here the results of some studies[30] of global parameter fits for the baryon octet and decuplet. The lagrangian used for this purpose (motivated by considerations of obtaining nonzero proton spin content from the skyrmion) is

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_{6,1} + \mathcal{L}_{6,2} + \mathcal{L}_{SB} \\
&= -\frac{F_\pi^2}{16}\mathrm{tr}(L_\mu L^\mu) + \frac{1}{32e^2}\mathrm{tr}[L_\mu, L_\nu]^2 \\
&- \epsilon_1 \frac{g_\omega^2}{m_\omega^2}\mathrm{tr}(\mathcal{B}^\mu \mathcal{B}_\mu) - \epsilon_2 \frac{g_\omega^2}{2m_\omega^2}\mathrm{tr}(\mathcal{B}^\mu)\mathrm{tr}(\mathcal{B}_\mu) \\
&+ \left[\frac{F_\pi^2}{32}(m_\pi^2 + m_\eta^2)\mathrm{tr}(U + U^\dagger - 2)\right. \\
&+ \left.\frac{\sqrt{3}F_\pi^2}{24}(m_\pi^2 - m_K^2)\mathrm{tr}(\lambda_8(U + U^\dagger))\right],
\end{aligned} \tag{129}$$

where, as earlier, $L_\mu \equiv U^\dagger \partial_\mu U$ and $\mathcal{B}_\mu$ is given by eq. (81). The resulting octet and decuplet masses are shown in Table 2, along with the deviation from experiment, which is about at the ten percent level or less. Octet magnetic

#	Params			M				dev
	e	F_π	ϵ_1	N/Δ	Λ/Σ	Σ/Ξ	Ξ/Ω	
1	6.0	82	-1.00	938	1111	1240	1366	98
				1226	1336	1485	1649	
2	6.0	74	-1.50	936	1114	1246	1375	120
				1234	1338	1469	1624	
3	6.0	89	-0.35	928	1124	1271	1402	131
				1233	1342	1488	1659	
4	6.6	82	-0.71	933	1108	1238	1364	99
				1249	1351	1480	1633	
5	5.4	82	-1.26	919	1119	1272	1410	159
				1203	1317	1469	1643	
6	8.0	186	-1.45	939	1120	1253	1387	120
				1236	1344	1476	1634	
exp				939	1116	1193	1318	
				1232	1385	1530	1672	

Table 2: Octet and decuplet masses in MeV. Members of octet in first line of entry; decuplet in second. Cases 1–5: $g_A \sim 1.24 \pm 0.05$; Case 6: $g_A = 1.72$.

moments are shown in Table 3 and are also surprisingly good. Last, we show in Table 4 a comparison with the proton spin content, which is also not unreasonable. Thus the application of skyrmions to the full SU(3) baryon octet and decuplet is generally quite successful.

8.3 YN and YY forces

The extension to SU(3) of the previous SU(2) methods based on the product ansatz and coupling to rotational and vibrational degrees of freedom shows similar amounts of attraction as were found before[112]. This extension also allows for the study[113,114] of ΣN, ΛN, and $\Lambda\Lambda$ interactions, to list only those for which there exist some rough data at present. (Since the product ansatz

#	μ							dev
	p	n	Λ	Σ^+	Σ^-	Ξ^0	Ξ^-	
1	2.65	-1.73	-0.83	2.80	-1.10	-1.75	-0.89	0.73
2	2.80	-1.77	-0.90	2.98	-1.12	-1.86	-0.99	0.93
3	2.54	-1.72	-0.75	2.64	-1.13	-1.65	-0.79	0.58
4	2.49	-1.57	-0.77	2.65	-1.02	-1.62	-0.88	0.69
5	2.87	-1.98	-0.88	3.01	-1.24	-1.90	-0.89	0.93
6	2.33	-1.76	-0.78	2.42	-0.97	-1.65	-0.65	0.68
exp	2.79	-1.91	-0.61	2.46	-1.16	-1.25	-0.65	

Table 3: Octet magnetic moments in units of μ_N

#	$1/\alpha^2$ [MeV]	$1/\beta^2$ [MeV]	$\Delta\Sigma$
1	130	536	0.17
2	132	615	0.34
3	130	452	0.04
4	145	574	0.14
5	115	473	0.17
6	130	810	0.58
exp			0.27 ± 0.13

Table 4: Proton spin content

is used, this approach does not allow for study of the H particle, which will be addressed briefly in the next subsection.) Once again central attraction is found if Roper vibrations are admixed, as for the NN case. The early work[113] in this sector was based on Monte Carlo methods for the projection onto baryon states with appropriate quantum numbers, which has an advantage of algebraic simplicity and flexibility, but inevitably carries with it the the need for long computer runs and the imprecision of the statistical method. Central attraction as strong or stronger than for the NN system was found.

The more accurate algebraic approach[114] found sizable attraction in systems with identical baryons, for example, NN and $\Lambda\Lambda$, when further symmetry breaking was introduced over and above that of eq. (122), i.e., when using $F_\pi \neq F_K$. This attraction increases with increasing strangeness content, so that at small interbaryon separation the central $\Xi\Xi$ potential is about four times more attractive than the NN one. The bulk of the attraction from this mechanism occurs at values of $R \leq 1$ fm, however, which reduces somewhat its relevance for conventional central baryon–baryon attraction. More recently, efforts are underway[115] to study YN interactions using stabilization mechanisms[75] that may allow for somewhat simpler calculational procedures.

8.4 The H-particle

An early suggestion by Jaffe[116], based on the MIT bag model, predicts the existence of a bound particle, called the H, with quantum numbers $J^\pi = 0^+$, $I = 0, Y = 0$ and a mass around 2150 MeV; this is a few tens of MeV below the $\Lambda\Lambda$ threshold for strong decay. In the quark model this particle, called the H, involves the six quarks $\{u, u, d, d, s, s\}$ in a fully-symmetric configuration. It is the object of experimental search (reviewed, e.g., in refs.[117,118]) to the present day. The possible existence of such a state was studied within the context of

skyrmion approaches almost from the beginning of this model[119,120,121,122] and also more recently[123]. The earlier work used the ansatz

$$U_H(\mathbf{r}) = \exp\left\{ i\hat{\mathbf{r}}\cdot\mathbf{\Lambda}\, f(r) + i\left[(\hat{\mathbf{r}}\cdot\mathbf{\Lambda})^2 - \frac{2}{3}\right]\, g(r)\right\}, \tag{130}$$

where $\mathbf{\Lambda} = \{\lambda_7, -\lambda_5, \lambda_2\}$ and $f(0) = g(0) = \pi$ and $f(r\to\infty) = g(r\to\infty) = 0$. More recently, the kaon bound-state ansatz of ref.[107] has been used[123] in this context. Unfortunately, the uncertainties in the skyrmion approach are sufficient so that it is difficult to reach an unambiguous statement as to whether the H is bound or not in this model. Worse still, other features such as the interaction with the continuum and vacuum effects (the Casimir energy—see subsection 4.4) make the situation even less clear.

9 The spin–orbit force

The study of the NN isoscalar spin–orbit force parallels in some respects the earlier work on the isoscalar central force: Lowest-order product-ansatz results again[70,124,125] show the wrong sign, that is, they find repulsion instead of attraction. Riska and Schwesinger[126] then suggested that the term $\mathcal{L}_{6,2}$ of eq. (80) would yield attraction, which indeed it does, but only at the cost of an unusually large coupling constant for this term[127]. The introduction of the dilaton[126] can enhance this attraction, as has been worked out more fully[128] using the lagrangian of eq. (116). In the course of carrying out such calculations one must deal with the "dragged" skyrmion, that is, the spin–orbit force is identified through the appearance of a term involving the motion of the baryon itself which must enter in the orbital angular momentum operator $\mathbf{L} = \mathbf{R}\times\mathbf{P}$ coupled to the spin $\boldsymbol{\sigma}$. All this comes about through a collective coordinate referring to the baryon position, in addition to the collective rotation.

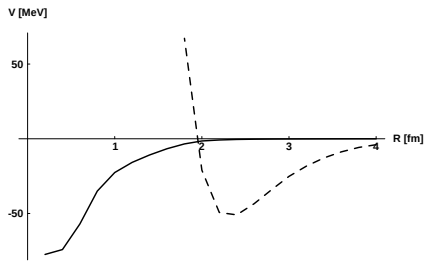

Figure 1: Spin-orbit potential and central potential (dashed line for $r \geq 2$ fm) for skyrmion parameters $F_\pi = 143$ MeV and $e = 20.0$, and dilaton parameters $\Gamma_0 = 137$ MeV and $C_G = (121\ \text{MeV})^4$, yielding a nucleon mass $M_N = 942$ MeV and a Δ mass $M_\Delta = 1174$ MeV. This result requires the presence of an $\mathcal{L}_{6,2}$ term and a dilaton.

One has

$$\dot{U}(\mathbf{r} - \mathbf{r}_k(t)) = A_k(t) \left\{ \frac{i}{2I}[\boldsymbol{\tau} \cdot \mathbf{j}_k,\, U_0(\mathbf{r} - \mathbf{r}_k)] - \frac{\partial_k}{M} \cdot \boldsymbol{\nabla} U_0(\mathbf{r} - \mathbf{r}_k) \right\} A_k^\dagger(t), \tag{131}$$

where M and I are the skyrmion mass and moment of inertia as hitherto.

Choosing the dilaton parameters $\Gamma_0 = 137$ MeV and $C_G = (121\ \text{MeV})^4$, with the empirical value for the ωNN coupling constant, $g_\omega^2/4\pi \sim 10$, sizable attraction is found[128], as seen in fig. 1, both for the central and the spin–orbit isoscalar components. These dilaton parameters imply a very low value for the glueball mass (~ 214 MeV) which is inconsistent with empirical expectations. If we take $\Gamma_0 = 61.2$ MeV and $C_G = (157.3\ \text{MeV})^4$ for a higher glueball mass of 809 MeV we get the results of fig. 2, with about an order of magnitude less

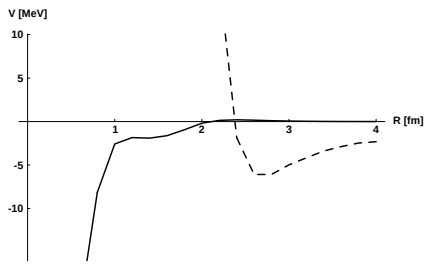

Figure 2: Spin-orbit potential and central potential as in fig. 1, but with dilaton parameters $\Gamma_0 = 61.2$ MeV and $C_G = (157.3 \text{ MeV})^4$ yielding a glueball mass of 809 MeV.

attraction, or about the same well depth as implied by vibrational admixtures (section 5.3). Since there are sizable technical difficulties[68,69,70,71,72,73,128,129] in considering the moving skyrmion—both at the computational level and in matters of principle—it would be premature to argue that the spin–orbit problem is solved. Indeed in addition to this special aspect of the spin–orbit force it also suffers from all the uncertainties noted for the more "static" central force. The particular choice of dilaton parameters also leads to a very low glueball mass. Although the dilaton mechanism itself is probably a well-motivated ingredient of skyrmion physics by virtue of the need to encompass the QCD trace anomaly, the appropriate values for the glueball mass involved are not clear and may well have to be substantially higher than the one used here (cp.[130].).

10 The nucleon in the nucleus

In this section we wish briefly to hint at possible applications of skyrmions to problems of a baryon in a medium which will become accessible once the two-baryon problem is under good control. The cases noted all relate to problems of active interest in nuclear physics at large. (We do not here address the large domain of problems involving skyrmion at nonzero temperatures which are of relevance to the study of hot hadronic matter and of the quark-gluon plasma.)

10.1 The $B = 3$ system

One of the characteristic features of the strongly-interacting many-body system—the nucleus—is the possibility of important contributions from three-body forces. These were early studied[131] using skyrmions, but without projection onto nucleon states through the methods discussed in eqs. (73)–(75). As a result, the forces found were very large ($\sim$ 100 or 200 MeV). Projection onto nucleon states immediately tames these forces, producing[132] potentials of 10

MeV or less, as one would have expected for three-nucleon forces. In addition to applications in systems of zero strangeness, the three-body force is also of interest in ΛNN systems[133], among others, for studies of the binding of hypernuclei.

10.2 Color transparency

There has been great interest in recent years (reviewed, e.g., in ref.[134]) in studying the phenomenon of color transparency, in which a nucleon is ejected from the nucleus with very high momentum transfer and experiences little final-state interaction as it exits: The high momentum transfer "catches" the nucleon in a small configuration, and, because the color charges of a small QCD object partially neutralize each other, the nucleon emerges while interacting weakly with the other nucleons. Thus color dynamics makes the nucleus transparent to the "small" nucleon (referred to here as a minisculon m). The crucial QCD ingredient in this process is precisely the mutual color neutralization, and thus it is of interest to examine this in QCD-based models. The skyrmion allows one to study this[135] by artificially shrinking the skyrmion and examining the consequent mN potential and total mN cross section. The shrinking is accomplished by using the scaling of eq. (52) and taking values for the minisculon such that its inverse size parameter $(eF_\pi)_m$ becomes large. The ratio of minisculon size to the size of a normal nucleon is then

$$x \equiv \frac{r_m}{r_N} = \frac{eF_\pi}{(eF_\pi)_m}. \tag{132}$$

The result of the skyrmion calculation using the product ansatz is that the central potential tends to zero more or less linearly with x for $x \leq 0.3$. On the other hand, the range of the potential tends to fall very gradually until $x \sim 0.05$ and then plunges rapidly to zero. The total NN cross section, in a

rough high-energy approximation, falls linearly to zero as x drops from unity to zero, showing quadratic behavior only when x is below 0.3.

10.3 Nucleon shrinking and swelling

There has been intense interest over the years[136,137,138,139,140,141,142] in a basic question of nucleon behavior in nuclei: Does the nucleon shrink or swell when placed in the nuclear medium? A related question is, Does the nucleon shrink or swell when in interaction with a second nucleon? Of course, if the skyrmion is used with the simple product ansatz the nucleon will surely shrink in a two-nucleon system, since we have already seen that this simple prescription leads to repulsion. However, when mechanisms for attraction are present, such as the admixture of higher baryon resonances (subsection 5.3) or the inclusion of a dilaton interaction (subsection 6), this situation changes, and one would expect a slight swelling for nucleons separated by 2 to 3 fm, followed by shrinking as they are brought closer together. This is indeed what is found in detailed calculation[143], as can be seen in figs. 3 and 4. The swelling effect is on the scale of 3 or 4 percent, which may be sufficient to explain present EMC results[144]; below about 1.5 fm the nucleons act to reduce each other's size. Such effects should show up as changes in the form factor of the nucleon within the nucleus for different ranges of momentum within the Fermi sea, and one may hope to see such effects in high-energy electron scattering, for example at CEBAF.

11 Summary

The first decade of studies of the two-baryon system with skyrmions has yielded an understanding of several different mechanisms that may contribute to central attraction and to the attractive isoscalar spin–orbit force. Thus the way is open to sort out, within the next few years, which of these effects are essential

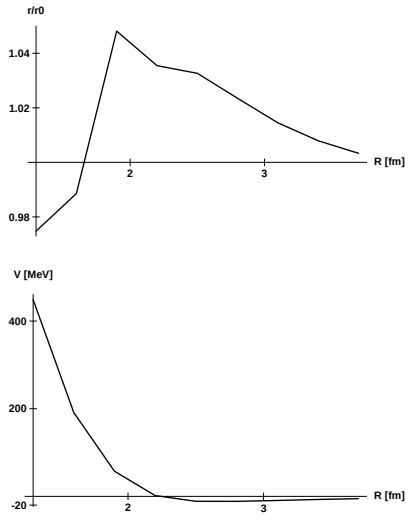

Figure 3: The NN potential and ratio of the radius for the interacting nucleon to that of the free nucleon r/r_0 for the usual skyrmion with product ansatz and baryon-resonance admixtures. (see subsections 4.3 and 5.3). The parameters are taken to have the values $F_\pi = 130$ MeV, $e = 20$, $\gamma = 0.50$, $\epsilon_{6,2} = 2.58$, and $m_\pi = 139$ MeV, yielding the masses $M_N = 998$ MeV, $M_\Delta = 1211$ MeV, and $M_{N*} = 1270$ MeV for the nucleon, Δ, and Roper.

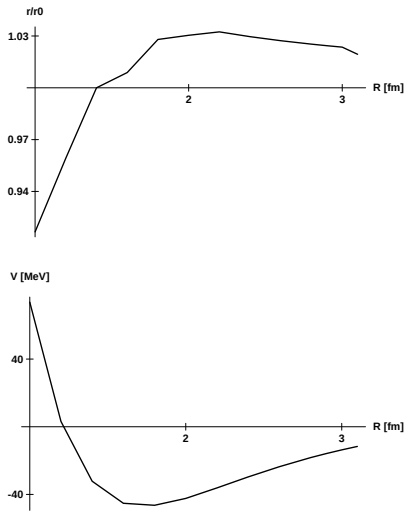

Figure 4: The NN potential and ratio of the radius for the interacting nucleon to that of the free nucleon r/r_0 for the skyrmion coupled to a dilaton. (see subsection 6). The parameters used were $\Gamma_0 = 306$ MeV, $C_G = (121 \text{ MeV})^4$, and $F_\pi = 186$ MeV, which produce $M_N = 1335$ MeV.

and dominant. It would be over-optimistic to claim that clear evidence for answers to that question exist today. In all likelihood, several of the candidates may enter together, thus strengthening the one overarching conclusion to be reached from QCD-related studies of the NN interaction so far, namely, that central attraction—however crucial it may be for nuclei to exist as we know them—is the result of an extremely subtle play of QCD effects.

It is a pleasure to thank José Sá Borges, Erasmo Ferreira, and Victoria Herscovitz for an invitation to deliver a series of lectures on skyrmions at Hadron Physics 96 in Brazil in April, 1996. We also acknowledge useful conversations on the subject matter of refs.[75,74,115] with Victoria Herscovitz, Juan Mignaco, and Gilberto Thomas, and regarding ref.[25] with Michel Betz. This work was supported in part by the Deutsche Forschungsgemeinschaft, the Israel Science Foundation, and the Yuval Ne'eman Chair at Tel Aviv University.

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