Back-reaction in the presence of thermalizing collisions

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Abstract

Preequilibrium parton production following an ultrarelativistic nucleus–nucleus collision is studied in terms of the decay of a strong chromoelectric field which generates pairs through the Schwinger mechanism. Back-reaction of the partons with the field is included and a model transport equation containing a collision term is solved for the central rapidity region based on an approximation in which the partons relax to a thermal distribution.

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1 Introduction

In recent years it has proved possible to solve the problem of back-reaction in the context of preequilibrium parton production in the quark–gluon plasma [1, 2]. This addresses the scenario in which two ultrarelativistic nuclei collide and generate color charges on each other, which in turn create a chromoelectric field between the receding disk-like nuclei. Parton pairs then tunnel out of this chromoelectric field through the Schwinger mechanism [3, 4, 5] and may eventually reach thermal equilibrium if the plasma conditions pertain for a sufficient length of time. While the tunneling and the thermalizing collisions proceed, the chromoelectric field accelerates the partons, producing a current which in turn modifies the field. This back-reaction may eventually set up plasma oscillations.

This picture for preequilibrium parton production has been studied in a transport formalism [6] in which the chromoelectric field is taken to be classical and abelian and collisions between the partons are completely ignored so that the only interaction of the partons is with the classical electric field, this interaction being the source of the back-reaction. Alternatively, the mutual scattering between partons has been considered in an approximation that assumes rapid thermalization and treats the collisions in a relaxation approximation about the thermal distribution [7]; in this study no back-reaction was allowed. Both interparton collisions and back-reaction were considered in a calculation [8] done in the hydrodynamic limit, which thus took into account only electric conduction within the parton plasma. All of these studies focused on the region of central rapidity.

The more recent calculations [1, 2] carried out a comparison between the transport formalism for back-reaction and the results of a field-theory calculation for the equivalent situation (see also [9, 10, 11]) and found a remarkable similarity between the quantal, field-theory results and those of the classical transport equations using a Schwinger source term. This link has also been established formally to a certain degree [12]. (This close relationship tends to fail, in part, for a system confined to a finite volume as a dimension of this volume becomes comparable with the reciprocal parton effective mass [13].) The studies relating field theory with transport formalism were all carried out under the assumption of a classical, abelian electric field and no parton–parton scattering. The removal of the assumption of a classical field, and thus the inclusion of interparticle scattering through the exchange of quanta, has been considered quite recently [14] for one spatial dimension.

The study reported here is carried out within the framework of the transport formalism (the parallel field-theory case is also currently under study [15]) and incorporates both back-reaction and a collision term in the approximation of relaxation to thermal equilibrium. Thus it assumes that thermalization takes place fast enough so that it makes sense to speak of the ongoing tunneling of partons, with back-reaction, as the collisions produce conditions of thermal equilibrium. It may be seen as combining the features of the studies of Białas and Czyż [6] with those of Kajantie and Matsui [7], or of paralleling the calculation [8] of Gatoff, Kerman, and Matsui, but at the level of the transport formalism without further appeal to hydrodynamics. Along with the other studies noted, it restricts itself to the region of central rapidity. The study provides a model for comparing the interplay between the thermalizing effects of particle collisions and the plasma oscillations produced by back-reaction.

2 Formalism

The transport formalism for back-reaction using boost-invariant variables has been presented previously in considerable detail [1] and is modified here only by the appearance of the collision term in the approximate form appropriate to relaxation to thermal equilibrium [7]. The Boltzmann–Vlasov equation in $3+1$ dimensions then reads, in the notation of [1],

$$p^\mu \frac{\partial f}{\partial q^\mu} - e p^\mu F_{\mu\nu} \frac{\partial f}{\partial p_\nu} = S + C, \tag{1}$$

where $f = f(q^\mu, p^\mu)$ is the distribution function, S is the Schwinger source term, and C is the relaxation-approximation collision term. The electromagnetic field is $F_{\mu\nu}$ and the electric charge e. The variables we take are

$$q^\mu = (\tau, x, y, \eta), \qquad p_\mu = (p_\tau, p_x, p_y, p_\eta), \tag{2}$$

where $\tau = \sqrt{t^2 - z^2}$ is the proper time and $\eta = \frac{1}{2}\log[(t+z)/(t-z)]$ is the rapidity. Thus, as usual, the ordinary, laboratory-frame coordinates are given by

$$z = \tau \sinh \eta, \qquad t = \tau \cosh \eta. \tag{3}$$

The momentum coordinates in eq. (2) relate to the laboratory momenta through

$$p_\tau = (Et - pz)/\tau, \qquad p_\eta = -Ez + tp, \tag{4}$$

where E is the energy and p is the z-component of the momentum, the z-axis having been taken parallel to the initial nucleus–nucleus collision direction or initial electric field direction.

Inserting expressions [1, 7] for the source term S and for the collision term C, and restricting to $1+1$ dimensions, the Boltzmann–Vlasov equation becomes

$$\begin{aligned} \frac{\partial f}{\partial \tau} + e\tau\mathcal{E}(\tau)\frac{\partial f}{\partial p_\eta} &= \pm(1 \pm 2f)e\tau|\mathcal{E}(\tau)| \log\left\{1 \pm \exp\left[-\frac{\pi m^2}{|e\mathcal{E}(\tau)|}\right]\right\}\delta(p_\eta) \\ &- \frac{f - f_{\rm eq}}{\tau_{\rm c}}. \end{aligned} \tag{5}$$

Here the upper sign refers throughout to boson production and the lower sign to the fermion case, and we have incorporated the necessary [1] boson enhancement and fermion blocking factor $(1 \pm 2f)$. The electric field is given for these variables by

$$\mathcal{E}(\tau) = \frac{F_{\eta\tau}}{\tau} = -\frac{1}{\tau}\frac{dA}{d\tau}, \tag{6}$$

where $A = A_\eta(\tau)$ is the only nonvanishing component of the electromagnetic four-vector potential in these coordinates. In eq. (5) we have assumed, as usual, that pairs emerge with vanishing p_η, which is the boost-invariant equivalent of the conventional assumption that pairs are produced with zero momentum in the laboratory frame.

The thermal equilibrium distribution is

$$f_{\rm eq}(p_\eta, \tau) = \frac{1}{\exp[p_\tau/T] \mp 1}, \tag{7}$$

where T is the system temperature, determined at each moment in proper time from the requirement [7]

$$\int \frac{dp_\eta}{2\pi}\, f(p_\eta, \tau)\, p_\tau = \int \frac{dp_\eta}{2\pi}\, f_{\rm eq}[T(\tau); p_\eta, \tau]\, p_\tau; \tag{8}$$

here and throughout $p_\tau = \sqrt{m^2 + p_\eta^2/\tau^2}$, where m is the parton effective mass, and the independent variables in terms of which the transport equations are evolved are p_η and τ. In eq. (5), $\tau_{\rm c}$ is the collision time or time for relaxation to thermal equilibrium.

Back-reaction generates variations in $\mathcal{E}(\tau)$ as a function of proper-time through the Maxwell equation

$$\begin{aligned} -\tau\frac{d\mathcal{E}}{d\tau} = j_\eta^{\rm cond} + j_\eta^{\rm pol} &= 2e\int \frac{dp_\eta}{2\pi\tau p_\tau}\, f\, p_\eta \\ &\pm [1\pm 2f(p_\eta=0,\tau)]\,\frac{me\tau}{\pi}{\rm sign}[\mathcal{E}(\tau)] \\ &\times \log\left\{1\pm\exp\left[-\frac{\pi m^2}{|e\mathcal{E}(\tau)|}\right]\right\}; \end{aligned} \tag{9}$$

here the two contributions on the right-hand side are for the conduction and polarization currents, respectively. Note that in $1+1$ dimensions the units of electric charge e and of the electric field $\mathcal{E}$ are both energy. For numerical convenience [1] a new variable is introduced, namely,

$$u = \log(m\tau), \qquad \tau = (1/m)\exp(u). \tag{10}$$

Equations (5) and (9) are to be solved as a system of partial differential equations in the independent variables p_η and τ for the dependent variables f and $\mathcal{E}$, determining the temperature T at each proper-time step from the consistency condition of eq. (8).

3 Numerical results and conclusions

The numerical procedures used here are patterned after those of ref. [13], and involve either the use of a Lax method or a method of characteristics. In practice the latter is considerably more efficient in this context and all results reported here are based on it. We note that these methods are completely different from those used in ref. [1]; as a check on numerical procedures we verified that full agreement was achieved with the results reported there. All quantities having dimensions of energy are scaled [1] here to units of the parton effective mass m, while quantities with dimensions of length are given in terms of the inverse of this quantity, $1/m$.

In order to present a relatively limited number of cases, we fix all our initial conditions at $u=-2$ in terms of the variable of eq. (10). At that point in proper time we take $\mathcal{E}=4$, with no partons present; the charge is set to $e=1$. This has been found [1] to be a rather representative case; in particular, little is changed by applying the initial conditions at $u=0$ rather than at $u=-2$. We shall exhibit results for three values of τ_c, namely 0.2, 1, and 10.

Our results are presented in fig. 1 for boson production and in fig. 2 for fermions. The uppermost graph in each case shows the temperature derived from the consistency condition of eq. (8) while the middle curves are for the electric field $\mathcal{E}$ and the lower graph gives the total currents. Both for bosons and for fermions, the cases with $\tau_c = 0.2$ and $\tau_c = 1$ involve a collision term that damps the distributions very rapidly. Thus no signs of plasma oscillations, which would arise if back-reaction came into play unhindered, are seen. For these values, the electric field and total current damp rather quickly to zero, and a fixed value of T is reached. The temperature peaks at around $1.5m$ for the boson cases, and near $2m$ for fermions. The temperature ultimately achieved depends, of course, on τ_c.

For $\tau_c = 10$, the plasma oscillations of back-reaction are clearly visible in the electric field and in the total current, both for bosons and for fermions. In fact, these cases are rather similar to their counterparts without thermalizing collisions [1], except for greater damping, especially of the current, when thermalization is involved. The plasma frequency is changed only a little by this damping. The plasma oscillations are reflected very slightly in the temperature behavior in a ripple at the onset of the oscillations, where they naturally have their largest excursion. However, the oscillations have the effect of pushing off the region at which a constant temperature is reached. Extending the calculation further out in the variable u, one finds that the temperature in that case levels off around $u \sim 5$ at a value of $T \sim 0.38$ for bosons and 0.39 for fermions.

In conclusion, this calculation allows an exploration of the transition between a domain dominated by parton collisions that bring about rapid thermalization in the quark–gluon plasma and a domain governed in major degree by back-reaction. In the first situation, the electric field from which the parton pairs tunnel, and the current which is produced from these pairs by acceleration in the field, both decay smoothly to zero and a terminal temperature is reached. In the latter case, plasma oscillations set in which delay somewhat the achievement of a final constant temperature. This qualitative difference between the two situations occurs for collision times about an order of magnitude larger than the reciprocal effective parton mass.

Note added in proof: After this paper was completed and posted in the Los Alamos archive, I learned of a similar study carried out by B. Banerjee, R.S. Bhalerao, and V. Ravishankar [Phys. Lett. B 224 (1989) 16]. The present work has several features that are different from the previous one, notably, the application to bosons as well as to fermions and the inclusion of factors for Bose–Einstein enhancement or Fermi–Dirac blocking in the Schwinger source term. By comparing with the field theory results, these factors have been found to be of considerable importance [1, 2, 10, 11]. The earlier work uses massless fermions, and, while the initial motion is taken to be one-dimensional as here, it includes a transverse momentum distribution, so that it is difficult to make a direct quantitative comparison between the two studies. There are also a number of technical differences between the calculations. Qualitatively, very similar behavior is found and the earlier study points out very clearly the necessity for treating the interplay between back-reaction and thermalization. I am very grateful to Professor R.S. Bhalerao for acquainting me with this earlier reference.

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References

[1] F. Cooper, J.M. Eisenberg, Y. Kluger, E. Mottola, and B. Svetitsky, Phys. Rev. D **48** (1993) 190.

[2] Reviewed in Y. Kluger, J.M. Eisenberg, and B. Svetitsky, Int. J. Mod. Phys. E **2** (1993) 333, where further references may be found.

[3] F. Sauter, Z. Phys. **69** (1931) 742.

[4] W. Heisenberg and H. Euler, Z. Phys. **98** (1936) 714.

[5] J. Schwinger, Phys. Rev. **82** (1951) 664.

[6] A. Białas and W. Czyż, Phys. Rev. D **30** (1984) 2371; **31** (1985) 198; Z. Phys. C **28** (1985) 255; Nucl. Phys. B **267** (1985) 242; Acta Phys. Pol. B**17** (1986) 635.

[7] K. Kajantie and T. Matsui, Phys. Lett. B **164** (1985) 373.

[8] G. Gatoff, A.K. Kerman, and T. Matsui, Phys. Rev. D **36** (1987) 114.

[9] F. Cooper and E. Mottola, Phys. Rev. D **40** (1989) 456.

[10] Y. Kluger, J.M. Eisenberg, B. Svetitsky, F. Cooper, and E. Mottola, Phys. Rev. Lett. **67** (1991) 2427.

[11] Y. Kluger, J.M. Eisenberg, B. Svetitsky, F. Cooper, and E. Mottola, Phys. Rev. D **45** (1992) 4659.

[12] Ch. Best and J.M. Eisenberg, Phys. Rev. D **47** (1993) 463.

[13] J.M. Eisenberg, Phys. Rev. D **51** (1995) 1938.

[14] F. Cooper, S. Habib, Y. Kluger, E. Mottola, J.P. Paz, and P.R. Anderson, Phys. Rev. D **50** (1994) 2848.

[15] J.M. Eisenberg, work in progress.

Figure captions:-

1. Temperature T, electric field $\mathcal{E}$, and total current j for the case of boson production. The curves are labeled with the values of the collision time or time for relaxation to thermal equilibrium τ_c introduced in eq. (5): $\tau_c = 0.2$, 1, and 10.

2. Same as fig. 1, but for fermion production.