Running coupling and anomalous dimension of SU(3) gauge theory with adjoint fermions

Thomas DeGrand[a], Yigal Shamir[b] and Benjamin Svetitsky[b]

Department of Physics, University of Colorado, Boulder, CO 80309, USA

[c]Raymond and Beverly Sackler School of Physics and Astronomy
Tel-Aviv University, Ramat Aviv, 69978 Israel

ABSTRACT

We propose to measure the running coupling and pseudoscalar renormalization constant of the SU(3) gauge theory with two flavors of adjoint fermions on 12^4 and 16^4 lattices. This will be combined with data from smaller lattices to give the beta function and mass anomalous dimension of the theory, which is a technicolor candidate. The total time requested is 3.3 million Jpsi-equivalent core-hours on the Fermilab clusters.

1 Physics goals

Gauge theories with groups other than SU(3), or with light fermions in representations higher than the fundamental, are a staple of theories that go beyond the Standard Model. Among the mechanisms proposed to connect these theories to the Standard Model at low energies are technicolor and tumbling, with many associated variants. Both of these depend largely on weak-coupling pictures for their dynamics: a perturbative β function to take technicolor from weak to strong coupling as the energy scale drops, and a most-attractive-channel argument for scale separation and selection of the condensed channel in tumbling. Nonperturbative tests of these pictures, long overdue, have appear only in the last four years—for reviews, see [1–3].

In the past four years we have carried out a program of lattice studies of theories with SU(N) gauge fields and fermions in the two-index symmetric representation of the gauge group [4–9]. A gauge theory's beta function can be negative (as in QCD) or positive (as in QED), or it can have a zero that is an infrared-attractive fixed point (IRFP). Theories with IRFPs are said to reside within a "conformal window" in the (N_c, N_f) plane, wherein the IR physics displays conformal invariance and no particle spectrum. Just outside the conformal window, there might be a borderland where the beta function approaches zero without actually crossing it; this would give candidates for "walking technicolor," where the running coupling comes to a near-standstill for many decades in the energy scale, until chiral symmetry breaking eventually sets in. See [4] for original references.

Our research so far has covered the SU(2), SU(3), and SU(4) gauge theory with two flavors of symmetric-representation fermions. These theories are believed to reside near the border of the conformal window. We calculated the nonperturbative beta function for each theory by means of a lattice version of the background field method, called the Schrödinger functional (SF). We here propose to extend our study to the SU(3) gauge theory with two flavors of fermions in the *adjoint* color representation.

Lattice studies of candidate theories for beyond - Standard Model physics applications, or more generally, theories with gauge theories with various numbers of colors and fermions in different representations, are a part of the broad USQCD program. The phenomenology of potential new nonperturbative physics is strongly constrained by precision electroweak measurements. What lattice simulations can do is to show whether theories which can evade these constraints and give new physics actually exist. While the field of lattice BSM studies has become fairly large, few groups other than us have done much work with fermions which are not in the fundamental representation. We are reasonably unique in possessing codes which allow efficient simulations for many different color and flavor representation possibilities. We regard our proposal as a very small "type A" proposal which addresses physics issues relevant to USQCD's strategic goals.

The direct determination of the beta function has proven quite difficult in all theories studied so far that are near the edge of the conformal window. These include the SU(3) theory with 12 fundamental fermions [10,11] as well as the three theories we have studied. In all these theories the perturbative beta function is small (compared, say, to QCD) precisely because the conformal window is nearby. All a numerical calculation can do is to confirm this property nonperturbatively; an actual zero of the beta function is very hard to dig out of the statistical error.

The very smallness of the beta function, on the other hand, makes it easy to determine another quantity to high precision: the anomalous dimension γ of the mass operator $\bar{\psi}\psi$, which is extracted from the scale dependence of the pseudoscalar renormalization constant Z_P. This anomalous dimension is crucial to the application of these gauge theories as theories of technicolor: $\gamma \simeq 1$ is necessary to prevent the elimination of a walking theory on the basis of flavor-changing neutral currents. Our calculations have yielded bounds of $\gamma < 0.5$ valid in the entire accessible ranges of the renormalized coupling, including the putative fixed points in each theory. Thus, whatever the eventual fate of the IR fixed point, these theories are close to being eliminated as technicolor candidates.

In all our studies we reaped the benefits of using Wilson-clover actions with fat links in the fermion operator. For the SU(2) theory, this enabled us to penetrate deep into strong coupling and uncover the fixed point convincingly. For SU(3) and SU(4), we were blocked by lattice artifacts until we supplemented the gauge coupling as well with fat-link plaquettes. This allowed us to move into strong coupling, find fixed points (with limited statistical significance), and show leveling off of the anomalous dimension well short of 1.

We proceed to the SU(3)/adjoint theory. Perturbation theory gives this theory a fixed point at a coupling that is a good deal weaker than in the theories studied so far. The goals of our project will be twofold: (1) to determine the beta function non-perturbatively; (2) to extract the anomalous dimension γ.

Calculations of the beta function and of the anomalous dimension proceed by studying the volume dependence of the running coupling and of Z_P, derived from appropriate observables. We will carry out calculations on volumes 12^4 and 16^4. Simulations of smaller volumes, 6^4 and 8^4, will be run elsewhere and will complete the picture.

Previous studies of the SU(3)/adjoint theory [12–15] have focused on its finite-temperature phase transitions. These studies discovered separated chiral and confinement transitions. They neglected the fact that the two-loop beta function predicts an IRFP at $g^2 \simeq 5.3$, and indeed they were limited to small values of N_t and thus did not verify continuum scaling of the transition temperature. Indeed, a theory with an IRFP will have a finite-temperature transition (or two) on a finite lattice, and it is only when N_t is made large enough that it will be seen to stall, rather than get pushed into weak coupling as is the case in QCD. Our experience with the symmetric-representation theories shows that study of the phase diagram alone may be misleading, and SF is a more suitable tool to tell conformality from confinement.

2 Computational strategy

We employ clover fermions with nHYP-smeared links [16]. Our gauge action will be a sum of the usual plaquette term and a plaquette term built of adjoint representation nHYP links, the so-called "soft action" of Ref. [9]. In our studies of $SU(3)$ and $SU(4)$ gauge theories coupled to symmetric-representation fermions, we encountered a strong coupling transition which basically wiped out the κ_c line. Replacing the Wilson gauge action by the soft action allowed us to push to stronger coupling. We will do the necessary tests of the gauge action, to see if we encounter the strong coupling transition and to tune the gauge action to avoid it, before we begin production running.

The scheme for computing the beta function is as follows: Working at fixed β we first vary κ until we find κ_c where the quark mass (defined through the axial Ward identity) is zero. We then measure the SF running coupling at the point $(\beta, \kappa_c(\beta))$ for several volumes L^4. The slope of $1/g^2$ vs. $\log L$ gives the beta function. Dropping the smallest latices or otherwise modeling finite-lattice effects gives an estimate of the systematic error due to the lattice spacing. The anomalous dimension γ is extracted from an inexpensive calculation of the pseudoscalar propagator from a wall source to a sink halfway across the lattice. This is done on the fly during the calculation of the SF coupling.

3 Software

We use the standard Hybrid Monte Carlo algorithm for generating gauge field configurations. We are using the MILC code [17], which we have modified to allow for varying the number of colors and for other representations besides the fundamental; we have also implemented the specific boundary conditions and observables needed for the SF calculation of the running coupling. We have implemented a number of technical improvements to the code. One is the use of a multiple-time-step [18] molecular-dynamics integrator along with Hasenbusch multimass preconditioning [19]. An additional modification is the streamlining of the conjugate gradient inversions through the use of lattice-wide field variables, significantly reducing the stride of memory accesses. Most recently, the adaptation to the adjoint theory included some modifications to take advantage of the reality of the representation.

We plan to run each 12^4 stream on 144 cores and each 16^4 stream on 512 nodes. In our study of the SU(3)/sextet theory we did some runs on `Jpsi` and on `Ds` and found good linearity in the number of nodes up to these values.

4 Resources requested

The time required should be the same as was needed for our study of the SU(3)/sextet theory. The gauge group is the same and the matrix-vector multiplications are almost the same: 8×8 real matrices multiplying complex vectors, instead of 6×6 complex matrices in the sextet case.

As in our SU(3)/sextet project we plan to measure the beta function and the anomalous dimension γ at six β values. We anticipate that this will include 3 values that are in fairly strong coupling, in the vicinity of the two-loop IRFP, and 3 at weaker couplings, to see the approach to the perturbative beta and gamma functions. We allocate twice as much time to each strong-coupling point to allow for slower convergence of conjugate-gradient inversions and also longer autocorrelations in the SF running coupling.

For the weak coupling points, adequate statistics on 12^4 lattices will require 70,000 core-hours each (times 3 points). Allowing twice as much for the three strong-coupling points brings us to a total of 630,000 core-hours for 12^4. To get to 16^4 we scale by L^5, where the factor of L is our empirical estimate for the increase in autocorrelations. Thus the 16^4 lattices will require 2.65 million core-hours. The total is thus 3.3 million core-hours

(`Jpsi` equivalent).

We are not requesting storage since our lattices are small and we can move them to our home facilities easily.

References

[1] G. T. Fleming, “Strong interactions for the LHC,” PoS **LATTICE2008**, 021 (2008) [arXiv:0812.2035 [hep-lat]].

[2] E. Pallante, “Strongly and slightly flavored gauge theories,” PoS **LAT2009**, 015 (2009) [arXiv:0912.5188 [hep-lat]].

[3] L. Del Debbio, “The conformal window on the lattice,” PoS LATTICE **2010**, 004 (2010).

[4] Y. Shamir, B. Svetitsky and T. DeGrand, “Zero of the discrete beta function in SU(3) lattice gauge theory with color sextet fermions,” Phys. Rev. D **78**, 031502 (2008) [arXiv:0803.1707 [hep-lat]].

[5] T. DeGrand, Y. Shamir and B. Svetitsky, “Phase structure of SU(3) gauge theory with two flavors of symmetric-representation fermions,” Phys. Rev. D **79**, 034501 (2009) [arXiv:0812.1427 [hep-lat]].

[6] T. DeGrand, Y. Shamir and B. Svetitsky, “Running coupling and mass anomalous dimension of SU(3) gauge theory with two flavors of symmetric-representation fermions,” Phys. Rev. D **82**, 054503 (2010) [arXiv:1006.0707 [hep-lat]].

[7] T. DeGrand, Y. Shamir and B. Svetitsky, “Infrared fixed point in SU(2) gauge theory with adjoint fermions,” Phys. Rev. D **83**, 074507 (2011) [arXiv:1102.2843 [hep-lat]].

[8] T. DeGrand, Y. Shamir and B. Svetitsky, “Mass anomalous dimension in sextet QCD,” arXiv:1201.0935 [hep-lat].

[9] T. DeGrand, Y. Shamir and B. Svetitsky, “SU(4) lattice gauge theory with decuplet fermions: Schrödinger functional analysis,” arXiv:1202.2675 [hep-lat].

[10] T. Appelquist, G. T. Fleming and E. T. Neil, “Lattice study of the conformal window in QCD-like theories,” Phys. Rev. Lett. **100**, 171607 (2008) [arXiv:0712.0609 [hep-ph]].

[11] T. Appelquist, G. T. Fleming and E. T. Neil, "Lattice study of conformal behavior in SU(3) Yang–Mills theories," Phys. Rev. D **79**, 076010 (2009) [arXiv:0901.3766 [hep-ph]].

[12] J. B. Kogut, J. Polonyi, H. W. Wyld and D. K. Sinclair, "Hierarchical Mass Scales In Lattice Gauge Theories With Dynamical Light Fermions," Phys. Rev. Lett. **54**, 1980 (1985).

[13] F. Karsch and M. Lutgemeier, "Deconfinement and chiral symmetry restoration in an SU(3) gauge theory with adjoint fermions," Nucl. Phys. B **550**, 449 (1999) [hep-lat/9812023].

[14] F. Basile, A. Pelissetto and E. Vicari, "The Finite-temperature chiral transition in QCD with adjoint fermions," JHEP **0502**, 044 (2005) [hep-th/0412026].

[15] J. Engels, S. Holtmann and T. Schulze, "Scaling and Goldstone effects in QCD with two flavors of adjoint quarks," Nucl. Phys. B **724**, 357 (2005) [hep-lat/0505008].

[16] A. Hasenfratz, R. Hoffmann and S. Schaefer, "Hypercubic smeared sinks for dynamical fermions," JHEP **0705**, 029 (2007) [arXiv:hep-lat/0702028].

[17] `http://www.physics.utah.edu/~detar/milc/`

[18] C. Urbach, K. Jansen, A. Shindler and U. Wenger, "HMC algorithm with multiple time scale integration and mass preconditioning," Comput. Phys. Commun. **174**, 87 (2006) [arXiv:hep-lat/0506011].

[19] M. Hasenbusch, "Speeding up the Hybrid-Monte-Carlo algorithm for dynamical fermions," Phys. Lett. B **519**, 177 (2001) [arXiv:hep-lat/0107019].