The running coupling in systems with 8 and 4+8 flavor and its implication for the spectrum

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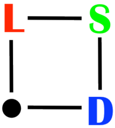
motivation

8 and 4+8 flavors

- Two projects based on the same action
  - SU(3) gauge group
  - Fundamental adjoint gauge action with $\beta_a = -\beta/4$ [Cheng et al. 2013][Cheng et al. 2014]
  - nHYP smeared staggered Fermions [Hasenfratz et al. 2007]
  - Most simulations/measurements performed with FUEL [J. Osborn]
- Common goals
  - Explore near conformal or conformal dynamics
  - Compute the iso-singlet 0^{++}
- Collaborators

8 flavor



+ Anna Hasenfratz

4+8 flavor

Richard Brower, Claudio Rebbi
Anna Hasenfratz, Evan Weinberg, OW

[JETP 120 (2015) 3, 423] [PoS Lattice2014 254]
[CCP proceedings 2014]

Lattice Strong Dynamics Collaboration

Argonne National Laboratory: Xiao-Yong Jin, James Osborn

University of California, Davis: Joe Kiskis

Oregon: Graham Kribs

Boston University: Richard Brower, Claudio Rebbi, Evan Weinberg

The University of Edinburgh: Oliver Witzel

RIKEN BNL Research Center: Ethan Neil, Sergey Syritsyn

Syracuse: David Schaich

Brookhaven National Laboratory: Meifeg Lin

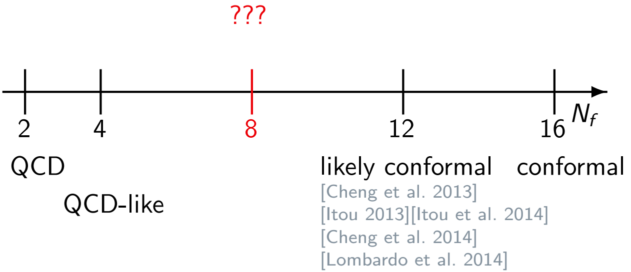
Colorado: Ethan Neil

Livermore: Evan Berkowitz, Mike Buchoff, Enrico Rinaldi, Chris Schroeder, Pavlos Vranas

Yale (Lux et Veritas): Tom Applequist, George Flemming

Motivation

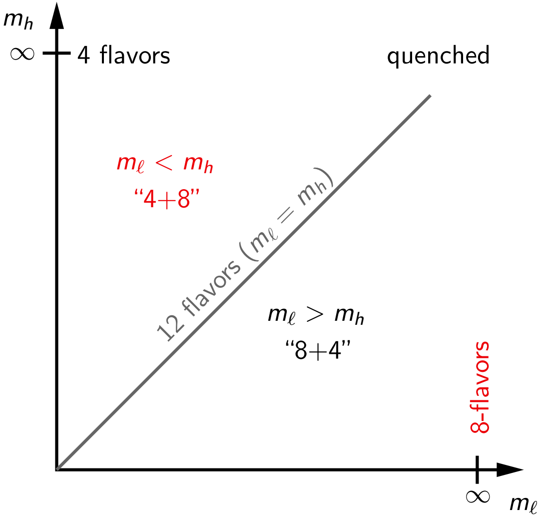
- SU(3) gauge theories with N_f fundamental fermions



- Staggered fermions come in multiplicities of 4 (no rooting)

$\Rightarrow$ 4, 8, 12, 16 are preferred N_f

Theories with $N_\ell = 4$ and $N_h = 8$ flavors



8 flavor vs. 4+8 flavors

8 flavors

- Most promising candidate for near conformal dynamics with SU(3) and integer number of fermions
- Interesting and important observations by LatKMI [Y. Aoki et al. 2014]
- Large scale resources required to explore chiral limit

4+8 flavors

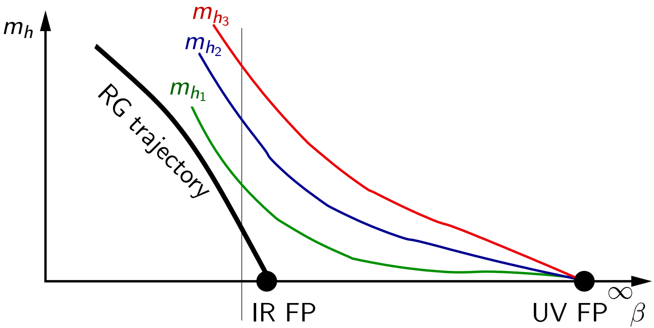
- General model to study near conformal behavior
- Heavy quark mass m_h is additional free, continuous parameter
- Sufficiently well known limits
  $m_h \to \infty$: 4-flavors
  $m_h \to m_\ell$: 12-flavors
- Has a continuum limit

$\Rightarrow$ Something interesting must happen

$\Rightarrow$ We can tune to be near the conformal window

Continuum limit in 4+8 flavors

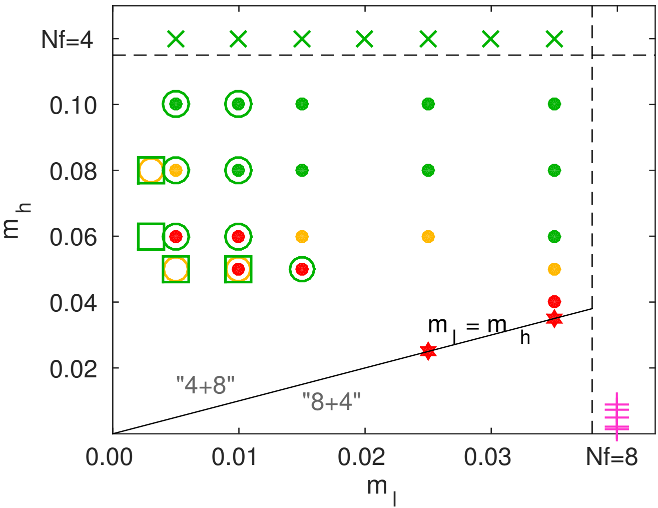
- We have 3 parameters: β, m_ℓ, and m_h
- First we take the chiral limit, i.e. $m_\ell \to 0$ and only 2 parameters remain
- Now we take the continuum limit by sending *together* $\beta \to \infty$ and $m_h \to 0$



- Practically this may be difficult and will require tuning

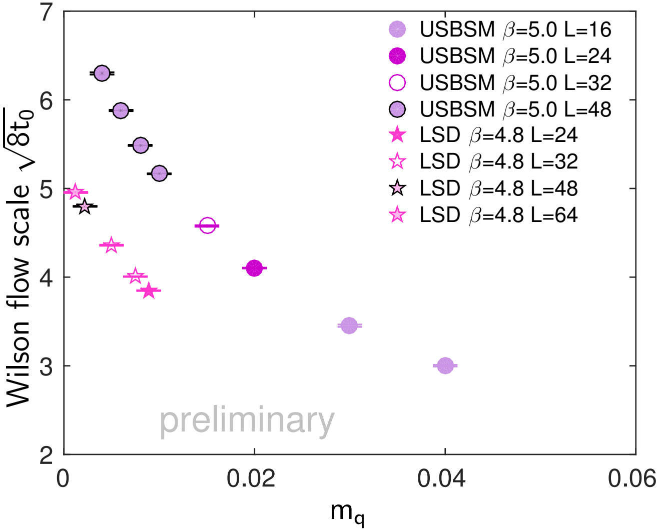
simulations

Performed simulations



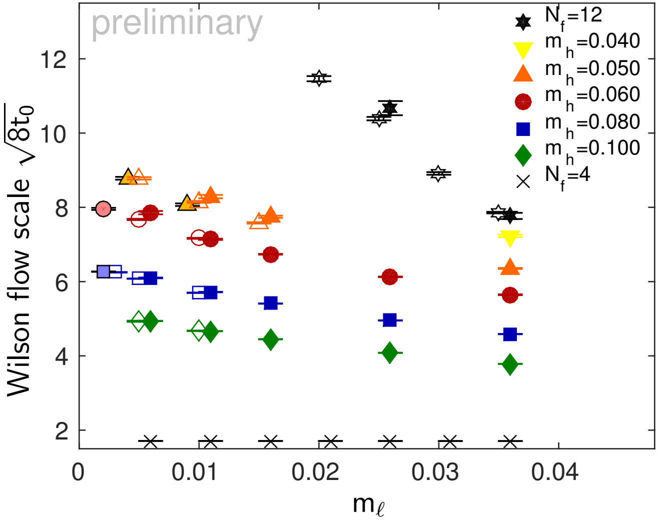
- 8 flavor simulations at $\beta = 4.8$ focus on chiral masses and are very expensive
- 4+8 simulations show many cheap ensembles on $24^3 \times 48$ lattice
- Symbols indicate volumes and colors finite volume effects

Lattice scales: 8 flavor



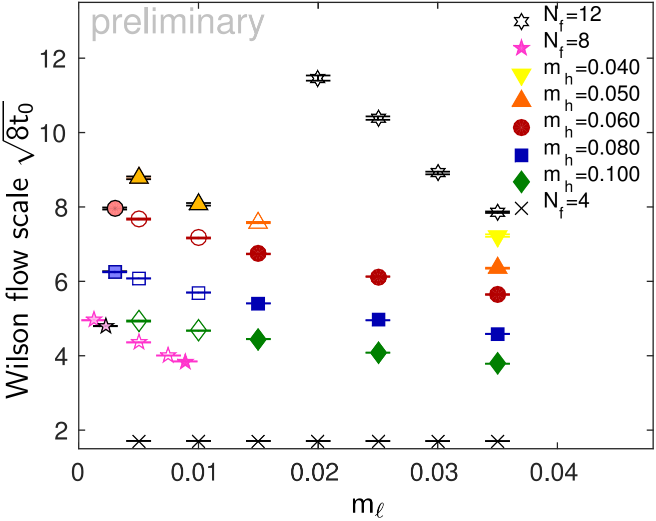
- Wilson flow scale $\sqrt{8t_0}$
- 8 flavors at $\beta = 4.8$ and $\beta = 5.0$
- USBSM:
  [Schaich, PoS Lattice2013 072]

Lattice scales



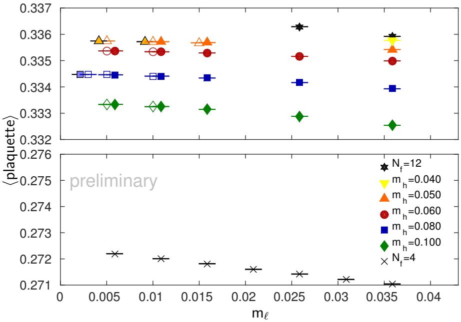
- Wilson flow scale $\sqrt{8t_0}$
- 4, 4+8, and 12 flavors at $\beta = 4.0$
- Data on 24^3, 36^3, and 48^3 volumes are shown small horizontal offset
- Small FV effects in the gauge sector

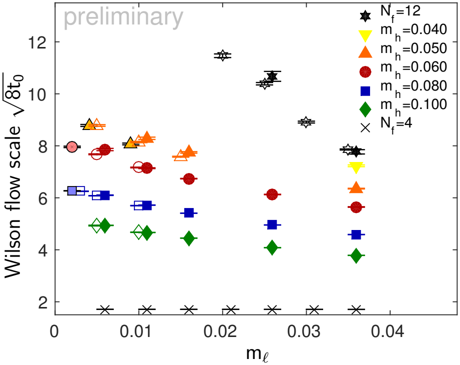
Lattice scales



- Wilson flow scale $\sqrt{8t_0}$
- 4, 4+8, and 12 flavors at $\beta = 4.0$
- 8 flavors at $\beta = 4.8$

Out of curiosity: the average plaquette



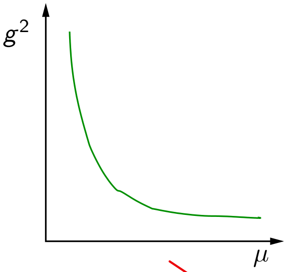


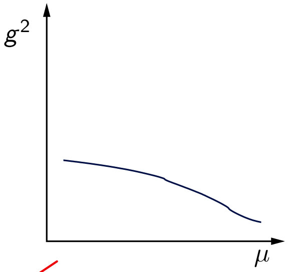
running coupling

Running coupling in 4+8

- 4 flavors:
  QCD-like fast running

- 12 flavors:
  flat, slow running induced by fermion mass





4+8 flavors??

- running coupling should change when varying m_h

Running coupling form gradient flow

- Gradient flow defines the renormalized coupling [Narayanan and Neuberger 2006] [Lüscher 2010]

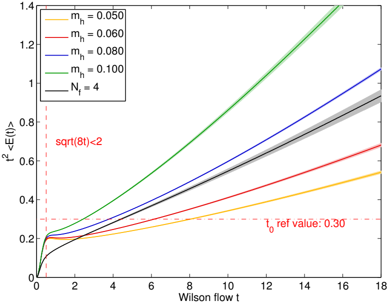
$$g^2_{GF}(\mu = 1/\sqrt{8t}) = t^2\langle E(t)\rangle/\mathcal{N}$$

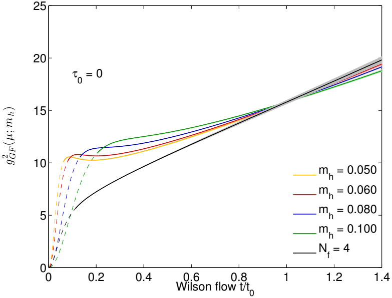
  t: flow time; $E(t)$ energy density
- g^2_{GF} is used for scale setting

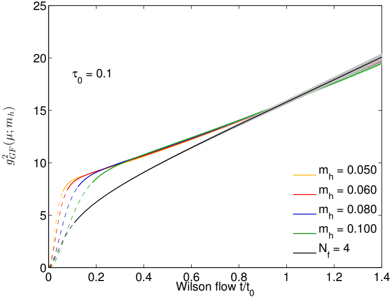
$$g^2_{GF}(t = t_0) = 0.3/\mathcal{N} \qquad (\text{“}t_0\text{-scale”})$$

- Can determine renormalized running coupling on large enough volumes and large enough flow times in the continuum limit

From $t^2\langle E(t)\rangle$ to the running coupling



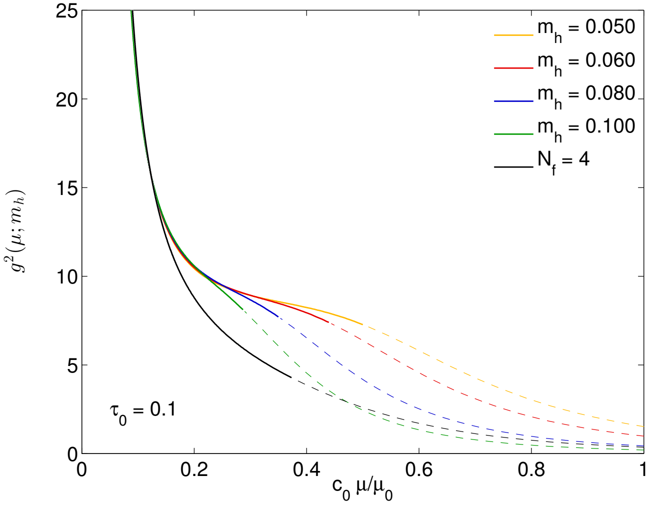




- Compute $t^2\langle E(t)\rangle$ as function of the flow time t
- Normalize by $\mathcal{N}$ and t_0
- Remove $O(a^2)$ errors by tau-shift

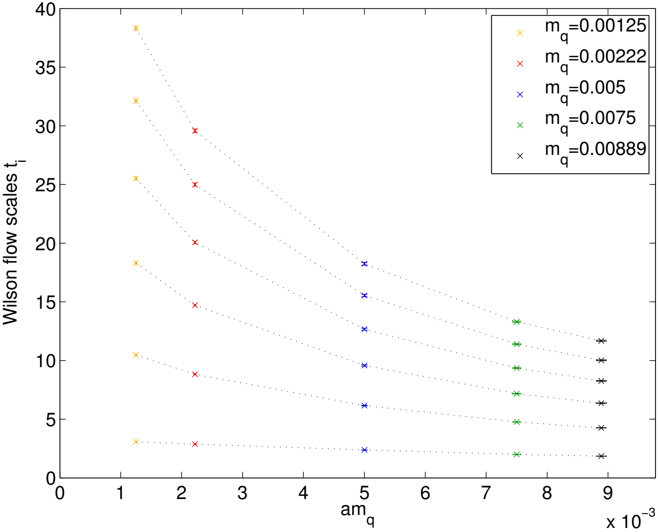
  $$g^2_{GF}(\mu) = \langle t^2 E(t+\tau_0 a^2)\rangle/\mathcal{N}$$
- Invert: $\mu = 1/t$

Running coupling form gradient flow: 4+8 flavors



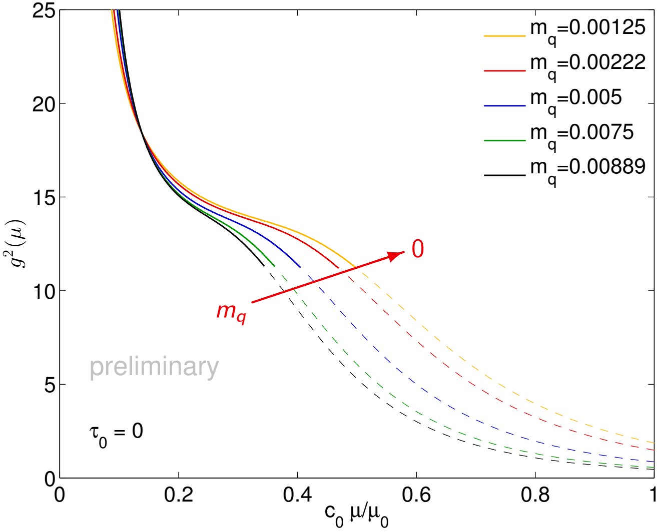
- Extrapolated to $m_\ell = 0$
- $N_f = 4$ shows fast running
- "Shoulder" increases for smaller m_h $\Rightarrow$ walking
- Walking range is tuned as function of m_h
- Data with error bars!

Running coupling form gradient flow: 8 flavors



- $\beta = 4.8$
- Chiral extrapolation is not feasible:
  - $\rightarrow$ non-linear chiral extrapolation
  - $\rightarrow$ no comparable data at different β
- Linear extrapolation observed in QCD and derived in χPT [Bär and Goltermann 2014]

Running coupling form gradient flow: 8 flavors



- Déjà vu?
- Is this walking?
- "Shoulder" extends as we lower m_ℓ
- Gauge dynamics causes the "shoulder": slow evolution
- Fermions trigger the fast rise
- What consequences arise form the two separate regimes?

anomalous dimension

Anomalous dimension

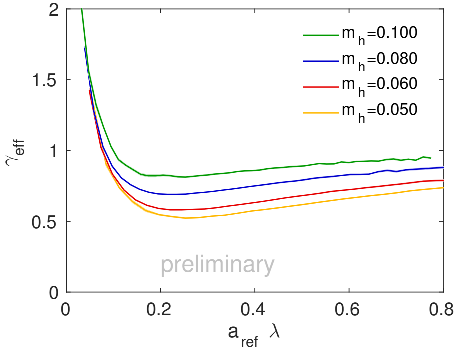
- We can predict a scale dependent anomalous dimension $\gamma_{\text{eff}}(\mu)$ form the mode number of the Dirac operator

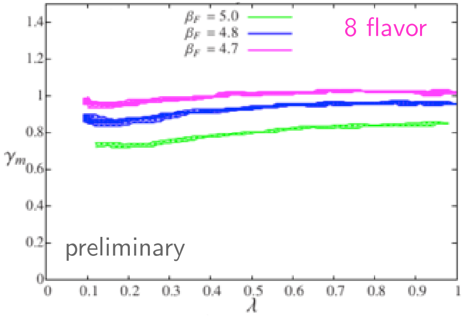
$$\mu(\lambda) \propto \lambda^{4/(\gamma_{\text{eff}}(\lambda)+1)} \quad \text{with} \quad \lambda \propto \mu$$

→ For large $\mu \sim \lambda$: $\gamma_{\text{eff}}(\mu)$ matches perturbative value

→ At $\lambda = 0$: $\gamma_{\text{eff}}(\mu)$ matches universal IRFP, if the system is conformal; meaningless once chiral symmetry breaks

Scale dependent anomalous dimension $\gamma_{\text{eff}}(\mu)$



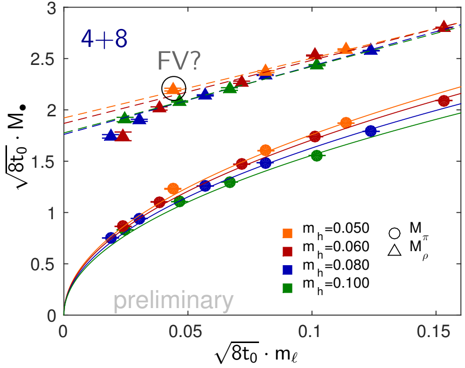


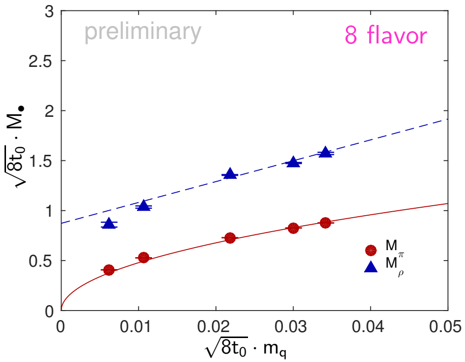
- Anomalous dimension is not large but still $O(1)$ and can persist
- For $m_h \to 0$ it approaches the value corresponding to the 12 flavor IRFP $\gamma^{\text{12f}}_{\text{IRFP}} = 0.235(15)$

- Anomalous dimension is around 0.9 for $\beta = 4.8$
- [Cheng et al. in preparation]

spectrum

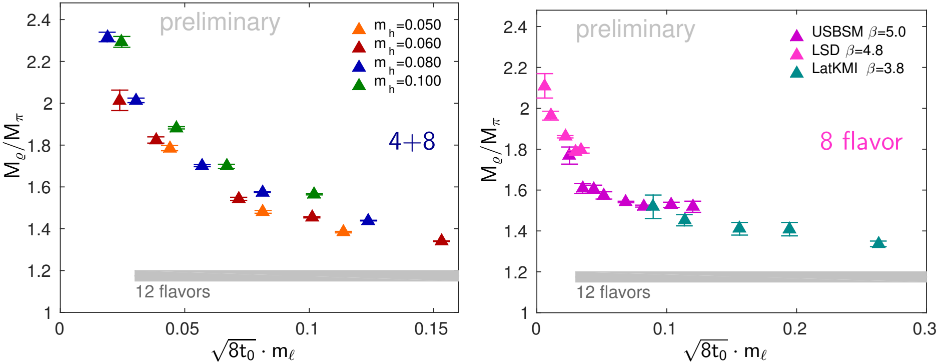
Connected spectrum





- Rescaling m_ℓ, m_q, M_π and M_ρ by $\sqrt{8t_0}$
- For 4+8 flavors we observe a weak dependence on m_h
- Fit lines intended "to guide the eye" assuming the naive expectation

Are we chirally broken?



- LatKMI data [Y. Aoki et al. 2014], USBSM data [Schaich, PoS Lattice2013 072]
- In 4 flavors (QCD) we know the ratio diverges
- In 12 flavors an almost constant ratio is observed [Cheng at al. 2014]

— as expected for conformal systems

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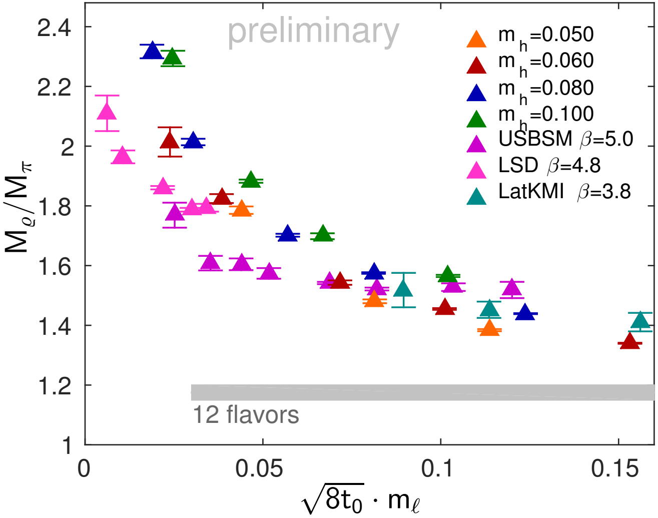

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Disconnected spectrum: the 0^{++} scalar

Both projects use the same setup

- 6 U(1) sources with dilution on each time slice, color and even/odd spatially
- Variance reduced $\langle\overline{\psi}\psi\rangle$

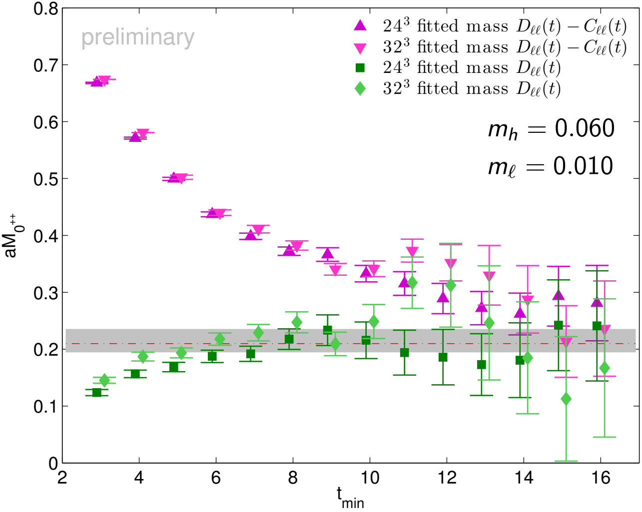
and the same analysis strategy

- Correlated fit to both parity states (staggered)
- Vacuum subtraction introduces very large uncertainties
- Advantageous to fit additional constant

$$C(t) = c_{0^{++}} \cosh\left(M_{0^{++}}\left(\frac{N_T}{2} - t\right)\right) + c_{\pi_{sc}}(-1)^t \cosh\left(M_{\pi_{sc}}\left(\frac{N_T}{2} - t\right)\right) + \nu$$

- Equivalent to fitting the finite difference: $C(t+1) - C(t)$

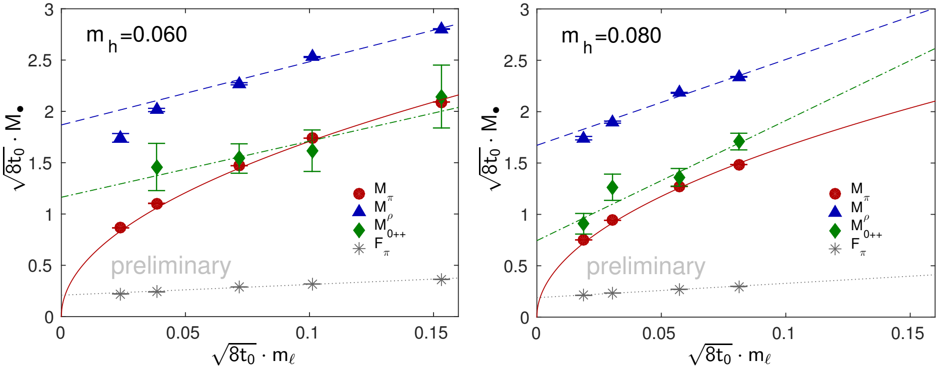
Comparison of $D_{\ell\ell}$ and $D_{\ell\ell} - C_{\ell\ell}$



- For $t \to \infty$: $D_{\ell\ell}$ and $D_\ell - C_{\ell\ell}$ should agree
- Compare fits with different t_{min} and $t_{\text{max}} = N_T/2$
- Compare results for two volumes

$\Rightarrow$ Consistent results!

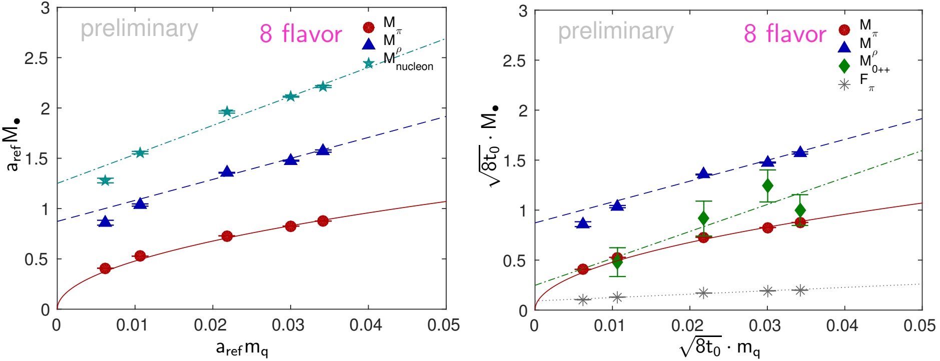
F_π, M_π, M_ρ, and $M_{0^{++}}$ for $m_h = 0.060$ and $m_h = 0.080$



- $m_\ell = 0: \ \sqrt{8t_0}M_\varrho = 1.87$,
  $\sqrt{8t_0}M_{0^{++}} = 1.16?$, $\sqrt{8t_0}F_\pi = 0.21$
- $m_\ell = 0.003: \ F_\pi L = 0.027 \cdot 48 = 1.3$

- $m_\ell = 0: \ \sqrt{8t_0}M_\varrho = 1.67$,
  $\sqrt{8t_0}M_{0^{++}} = 0.74$, $\sqrt{8t_0}F_\pi = 0.19$
- $m_\ell = 0.003: \ F_\pi L = 0.034 \cdot 36 = 1.2$

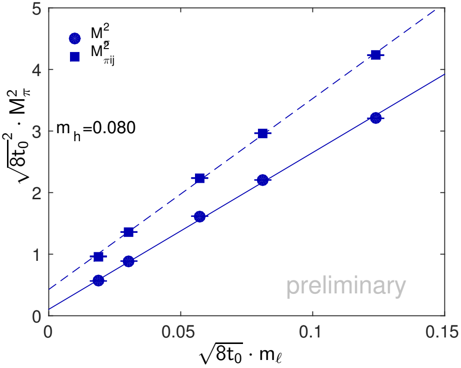
F_π, M_π, M_ρ, M_{nucleon} and $M_{0^{++}}$ for 8 flavors

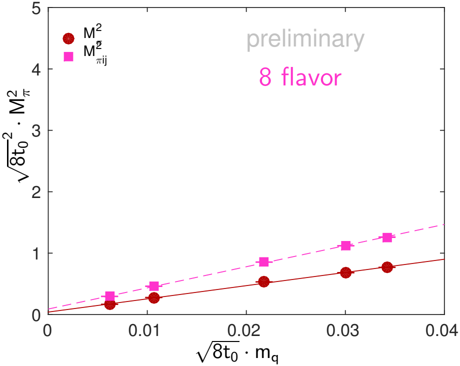


- $m_\ell = 0$: $\sqrt{8t_0}M_\varrho = 0.87$, $\sqrt{8t_0}M_{\text{nucleon}} = 1.25$, $\sqrt{8t_0}M_{0^{++}} = 0.25$, $\sqrt{8t_0}F_\pi = 0.09$
- $m_\ell = 0.00222$: $F_\pi L = 0.027 \cdot 48 = 1.3$
- Connected spectrum not too happy with "naive assumptions for fit"

Pion taste splitting

- Taste splitting is artifact of staggered fermions
- In QCD modern, smeared staggered actions show small taste splitting effects
- Taste splitting is typically constant w.r.t. m_q





- In 4+8 splitting increases for larger m_ℓ when reducing m_h

remarks

Concluding remarks

4+8 flavors

- A great model to explore near conformal dynamics by varying the continuous parameter m_h
- Limiting cases of 4 and 12 flavors help to understand what is happening

8 flavors

- A very difficult system requiring very expensive simulations to investigate
- May be very close to the onset of the conformal window

Non-QCD like features

- Running coupling develops a “shoulder”
- Chiral behavior can be tuned with m_h
- Regime of slower running coupling
- Chiral behavior only visible for very small bare fermion masses
- Curvature of M_ϱ
- Non-constant taste splitting
- The 0^{++} is light: $M_{0^{++}} < M_\varrho, 2M_\pi$