

# Strongly Interacting Gauge Theory and Physics Beyond Standard Model

## Mass Anomalous Dimension $\gamma$

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# Motivation

- The Higgs boson with  $M_H \simeq 125$  GeV is discovered (2012, LHC-CERN)!
- The LHC second-run has started!

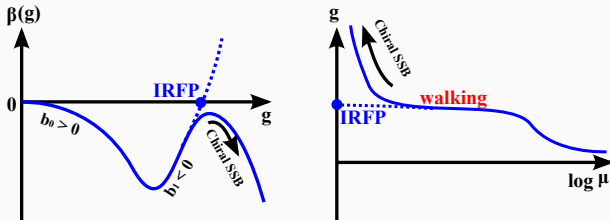
Why Not Investigate The Origin of EWSB?

# Strong Dynamics: Origin of EWSB

## Questions/Answers: New Strong Dynamics

- 1 Physical Contents of Higgs?  
Composite Higgs(c.f.  $\sigma$ , Cooper-Pair).
- 2 Physics of Electroweak (EW) Symmetry Breaking?  
Chiral Symmetry Breaking.
- 3 Fine-Tuning Problem for  $M_H = 125$  GeV?  
Log (partly Power-Low) corrections for  $M_H = 125$  GeV.

# Many Flavor QCD and Walking



- Beta function of  $N_f$  flavor in  $SU(N_c = 3)$  gauge theory:  

$$\beta(g) = dg(\mu)/d\mu = -b_0(N_f, N_c)g^3 - b_1(N_f, N_c)g^5 + \dots$$
- In  $8 \lesssim N_f \lesssim 12$ , **Walking Dynamics**  
 (Schwinger Dyson Analyses, Yamawaki et.al.('86), Holdom ('85)).
- Stable (light) Higgs: **Techni-Dilaton (PNGB for Scale Sym Breaking)**.
- SM-lepton/quark masses: **Enhancement by Factor  $(\Lambda_{ETC}/\Lambda_{EW})^\gamma$**

## Subject: Mass Anomalous Dimension

We investigate the mass anomalous dimension  $\gamma$  in many flavor QCD by using

- Lattice Gauge Theory with Monte Carlo Simulations  
(Latest 8-Flavor Results in LatKMI Collaboration).
- Schwinger-Dyson (SD) Equations  
(Miura-Nagai-Shibata, UV/IR Cutoff Effects).

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- 2 8-flavor QCD with regards to WTC (LatKMI, Preliminary)
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  - Some Introduction
  - IR Cutoff Effects
- 4 Summary and Future Perspective

# LatKMI Collaboration

Y. Aoki	(KMI, Nagoya Univ)
T. Aoyama	(KMI, Nagoya Univ)
E. Bennett	(Swansea Univ (UK))
M. Kurachi	(KEK)
T. Maskawa	(KMI, Nagoya Univ)
K. Miura	(CPT, Aix-Marseille Univ (RF) / KMI, Nagoya-Univ)
K-i. Nagai	(KMI, Nagoya Univ)
H. Ohki	(RIKEN)
T. Yamazaki	(Tsukuba Univ)
K. Yamawaki	(KMI, Nagoya Univ)
E. Rinaldi	(LLNL (US))
A. Shibata	(KEK)

# Setups I

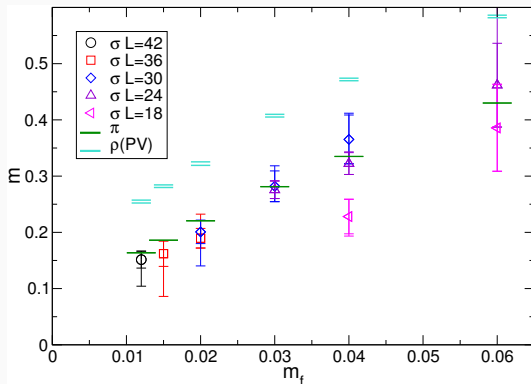
- Lattice Action:**  $N_f = 8$  HISQ Action  
+ Tree-level Symanzik Gauge Action with  $\beta = 3.8$ .
- Algorithm:** HMC with Hasenbush pre-conditioning.
- Observables:**  $F_\pi$ ,  $M_\pi$ ,  $M_\rho$ ,  $M_{a1}$ ,  $M_N$ , VPF,  $\dots$ .
- Code etc:** MILC ver.7.6.3 with some modifications, SciDac Library.
- Computer:** KMI HPC Cluster  $\varphi$ ,  
Nagoya-Univ-ITC CX400,  
Kyushu-Univ-RIIT CX400/HA8000.



## Setups II

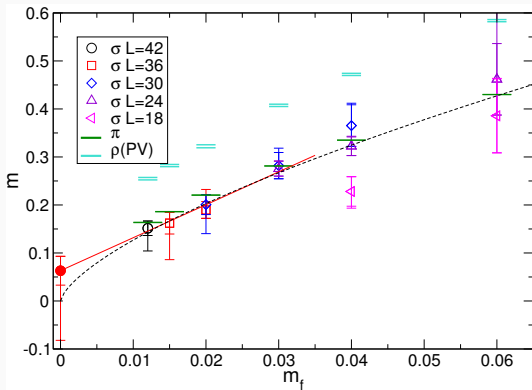
★: New Configs.    ★: Updates,  $\mathcal{O}(10^4)$  Configs.

$m_f \setminus L$	42	36	30	24	18	12
0.012	★					
0.015	★	★				
0.02		★	★	★		
0.03		★	★	★		
0.04			★	★	★	
0.05			★	★	★	★
0.06			★	★	★	
0.07			★	★	★	★
0.08				★	★	★
0.09						★
0.10				★	★	★
0.12						★
0.14						★
0.16						★

$N_f = 8$  Flavor Singlet Scalar  $\sigma$  I (Update from LatKMI PRD 2014)

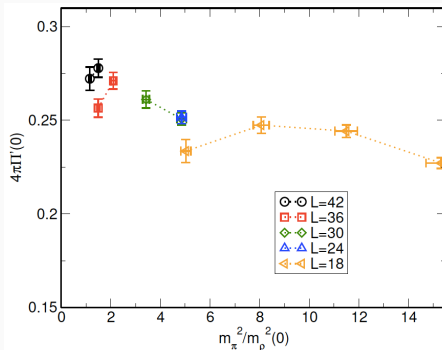
Light  $\sigma \sim$  Dilaton (PNGB for Broken Scale Symm.)

# $N_f = 8$ Flavor Singlet Scalar $\sigma$ II (Update from LatKMI PRD 2014)



Light  $\sigma \sim$  SM Higgs (125 GeV)?

(c.f. LatHC Collab. ('14), Hietanen et.al. ('14), Athenodorou et.al. ('15)).

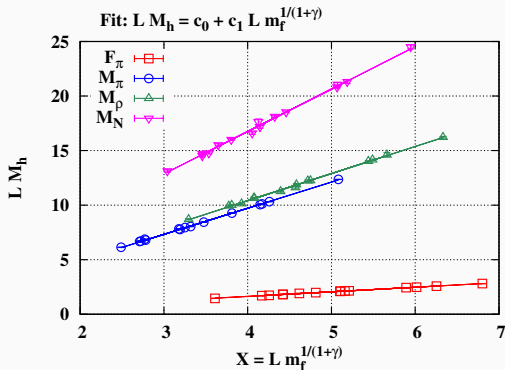
$N_f = 8$  S-Parameter

$$S \equiv 4\pi\Pi'_{V-A}(Q^2 \rightarrow 0)$$



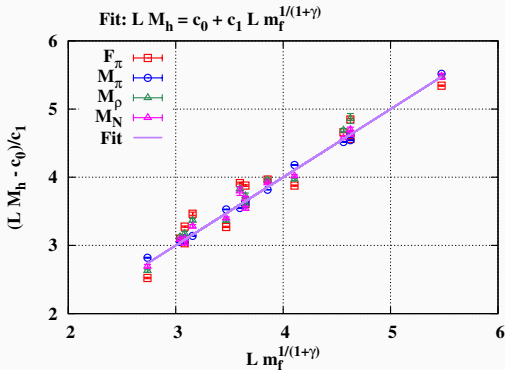
- The result  $S \sim 0.25 - 0.275$  is smaller than that  $S_{\text{QCD}, N_f=2} \sim 0.43$ .
- Walking  $\sim$  Weaker Chiral SSB  $\sim$  V-A Doubling  
(c.f. LSD-Collab. ('14), Knecht et.al. (Large  $N_c$  '98)).
- Still much larger than  $S_{\text{exp}} = 0.03(10)$ .

# FSHS: Individual Fits



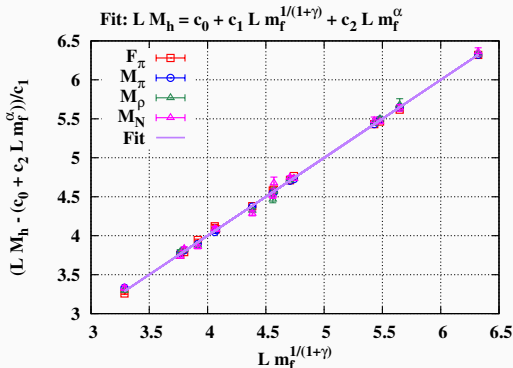
	$F_\pi$	$M_\pi$	$M_\rho$	$M_N$
$\gamma$	1.003(5)	0.627(2)	0.896(11)	0.810(11)
$\chi^2/\text{dof}$	2.34	15.26	1.41	2.58

## FSHS: Simultaneous Fits



$$(\gamma, \chi^2/\text{dof}) = (0.709(2), 124.96).$$

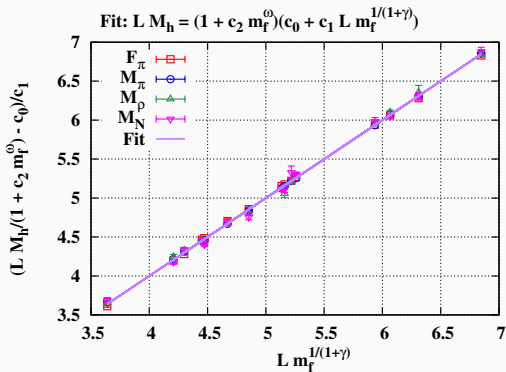
# FSHS: Simultaneous Fits with Mass Collection



$$(\gamma, \chi^2/\text{dof}) = (0.893(15), 3.26) \text{ for } \alpha = 1.$$

# FSHS: Simultaneous Fits with A-Hasenfratz Type Collection

Fit Ansatz: Cheng-Hasenfratz-Liu ('14).

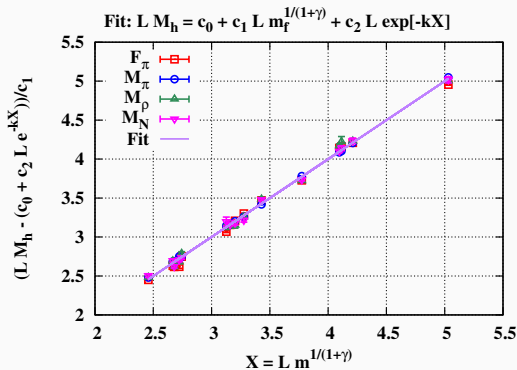


$$(\gamma, \chi^2/\text{dof}) = (1.014(35), 2.46) \text{ for } \omega = 0.35.$$



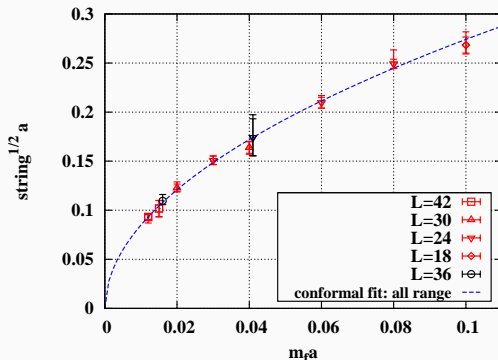
# FSHS: Simultaneous Fits with Frascati-Groningen Type Collection

Fit Ansatz: Lombardo-Miura-Silva-Pallante ('14).



$$(\gamma, \chi^2/\text{dof}) = (0.617(2), 12.80) \text{ for } k = 0.1.$$

# $N_f = 8$ String Tension



$$\sqrt{\text{string}} \cdot a = C(m_f a)^{1/(1+\gamma)},$$

$$(\gamma, \chi^2/\text{dof}) = (0.97(4), 0.68).$$

## Summary on $N_f = 8$ QCD

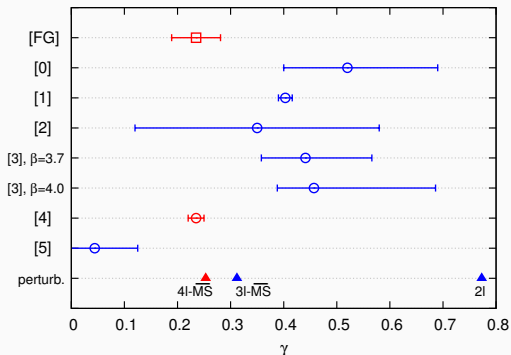
FSHS Individual	$F_\pi$	$M_\pi$	$M_\rho$	$M_N$
$\gamma$	1.003(5)	0.627(2)	0.896(11)	0.810(11)
$\chi^2/\text{dof}$	2.34	15.26	1.41	2.58

FSHS Simultaneous	$\gamma$	$\chi^2/\text{dof}$
$LM_h = c_0 + c_1 X, X = Lm_f^{1/(1+\gamma)}$	0.709(2)	124.96
$LM_h = c_0 + c_1 X + c_2 Lm_f^{\alpha=1}$	<b>0.893(15)</b>	<b>3.26</b>
$LM_h = c_0 + c_1 X + c_2 Lm_f^{\alpha=2}$	0.772(5)	19.38
$LM_h = (1 + c_2 m_f^{\omega=0.35})(c_0 + c_1 X)$	<b>1.014(35)</b>	<b>2.46</b>
$LM_h = c_0 + c_1 X + c_2 L \exp(-kX) _{k=0.1}$	0.617(2)	12.80

- For  $m_f \in [0.012, 0.03]$ , the quadratic fit (motivated by ChPT) works for all measured operators. However,  $N_f(M_\pi/(4\pi F/\sqrt{2}))^2 \gtrsim 6$  and  $FL \lesssim 0.85$ .
- For the same  $m_f$  range, the naive hyper-scaling fit also works except few cases with somewhat larger  $\chi^2/\text{dof}$ . The  $\gamma$  is operator dependent.

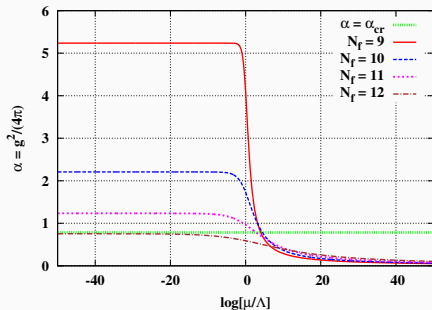
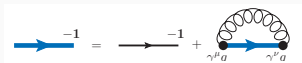
$N_f = 8$  QCD: Having Light  $\sigma$ , Showing Quasi-Conformal Nature with  $\gamma \sim 1$ .

# Motivation: Lattice Results on $\gamma$ in Color $SU(N_c = 3)$ with $N_f = 12$

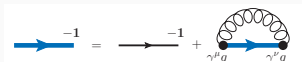


[FG] Nunes et.al. ('14), [0] Deuzemann et.al. ('09), [1] LSD-Collab ('11), (using data provided in [7]), [2] DeGrand ('11), (using data provided in [7]), [3] LatKMI ('12), [4] Cheng et.al. ('13), [5] Ito ('13), [6] Shrock ('13) [7] Fodor et.al. ('11).

# SD with Too-Loop Improved Ladder Approx.



- $N_f \geq 8.05$ : Banks-Zaks (BZ) Infra-Red Fixed Point (IRFP,  $\alpha_*$ ).
- $N_f \gtrsim 12$ : Conformal Window with  $\alpha_* \leq \alpha_{\text{cr}} = \pi/(3C_2[F])$ .

SD with  $\Lambda_{UV/IR}$ 

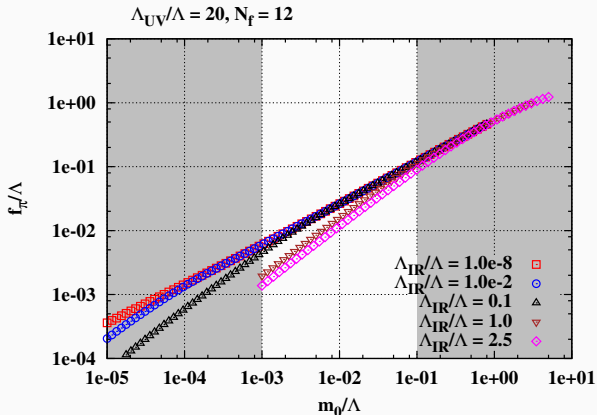
$$iS_F^{-1}(q) = [A(p^2)\not{p} - \Sigma(p^2)] \text{ (Fermion Invers-Prop.) ,}$$

$$m_p(m_0, \Lambda_{UV/IR}) = \Sigma(p^2 = m_p; m_0, \Lambda_{UV/IR}) \text{ , (Pole Mass, SD-Output) ,}$$

$$f_\pi^2(m_0, \Lambda_{UV/IR}) = \frac{N_c}{4\pi^2} \int dz z \frac{(1 - \frac{1}{4}z \frac{d}{dz})\Sigma^2(z)}{(z + \Sigma^2(z))^2} \text{ , (Pagels-Stoker) .}$$

## State of The Arts (c.f. Previous Work (LatKMI '09))

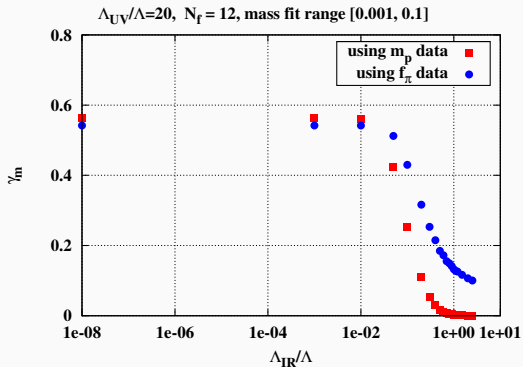
- The full momentum dependence of the two-loop running coupling  $g$  ( $\Lambda_{UV/IR}$  vs scales encoded in  $g$ ).
- Wide range probe in the parameter space ( $\Lambda_{IR}/\Lambda \in [10^{-8}, 2.5]$  and  $\Lambda_{UV}/\Lambda \in [1.0, 20]$  including  $f_\pi/\Lambda_{IR} \sim f_\pi L \sim 1$ ).

$N_f = 12$   $f_\pi$  vs  $m_0$ 

$$M_h = C m_0^{1/(1+\gamma_m)}, \quad M_h = m_p \text{ or } f_\pi. \quad (1)$$

$N_f = 12 \quad \gamma_m \text{ vs } \Lambda_{\text{IR}}$ 

$$M_h = C m_0^{1/(1+\gamma_m)}, \quad M_h = m_p \text{ or } f_\pi. \quad (2)$$





## SD-Based FSHS: Formulation

$$\frac{m_0}{Z_{UV} m_p} = \frac{A_{\omega_m}(y_{IR}) D_{\omega_m}(y_{\Lambda}) \left(\frac{1+y_{IR}}{1+y_{\Lambda}}\right)^{(1-\omega_m)/2} + (\omega_m \leftrightarrow -\omega_m)}{A_{\omega_m}(y_{IR}) N_{\omega_m}(\max\{1, y_{IR}\}) \left(\frac{1+y_{IR}}{1+\max\{1, y_{IR}\}}\right)^{(1-\omega_m)/2} + (\omega_m \leftrightarrow -\omega_m)},$$

$$A_{\omega_m}(y) = \frac{1 + \omega_m}{2\omega_m} F\left[\frac{-1 + \omega_m}{2}, \frac{-1 + \omega_m}{2}, 1 + \omega_m; \frac{1}{1+y}\right],$$

$$D_{\omega_m}(y) = \frac{1 + \omega_m}{2} F\left[\frac{1 - \omega_m}{2}, \frac{1 - \omega_m}{2}, 1 - \omega_m; \frac{1}{1+y}\right],$$

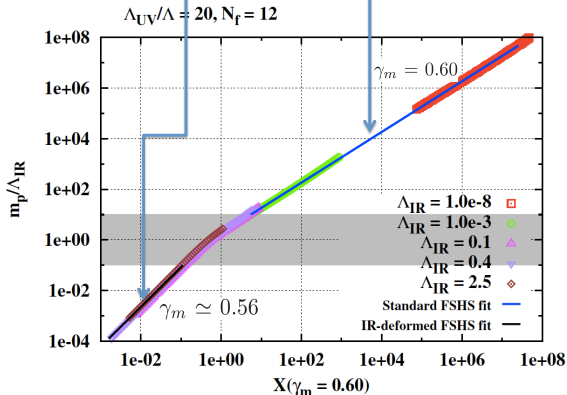
$$N_{\omega_m}(y) = F\left[\frac{1 - \omega_m}{2}, \frac{3 - \omega_m}{2}, 1 - \omega_m; \frac{1}{1+y}\right], \quad (3)$$

$$\omega_m = 1 - \gamma_m, \quad (y_{IR}, y_{\Lambda}) = \left(\frac{\Lambda_{IR}^2}{m_p^2}, \frac{\Lambda^2}{m_p^2}\right). \quad (4)$$

$$\frac{\hat{m}_p}{\hat{\Lambda}_{IR}} = \begin{cases} C_X(\gamma_m, Z_m^{UV}) \cdot X, & X \equiv \hat{m}_0^{1/(1+\gamma_m)} / \hat{\Lambda}_{IR} \quad (\Lambda_{IR} \ll m_p \ll \Lambda), \\ C_Y(\gamma_m, Z_m^{UV}) \cdot Y, & Y \equiv \hat{m}_0 / \hat{\Lambda}_{IR}^{-(1+\gamma_m)} \quad (m_p \ll \Lambda_{IR} \ll \Lambda). \end{cases} \quad (5)$$

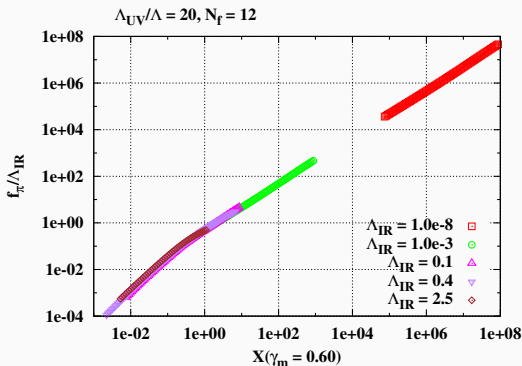
# $N_f = 12$ SD-Based FSHS I

$$\frac{\hat{m}_p}{\hat{\Lambda}_{\text{IR}}} = \begin{cases} C_X(\gamma_m, Z_m^{\text{UV}}) \cdot X \\ C_Y(\gamma_m, Z_m^{\text{UV}}) \cdot Y \end{cases}$$



## $N_f = 12$ SD-Based FSHS II

$$\frac{\hat{m}_p}{\hat{\Lambda}_{\text{IR}}} = \begin{cases} C_X(\gamma_m, Z_m^{\text{UV}}) \cdot X, & X \equiv \hat{m}_0^{1/(1+\gamma_m)} / \hat{\Lambda}_{\text{IR}} \quad (\Lambda_{\text{IR}} \ll m_p \ll \Lambda), \\ C_Y(\gamma_m, Z_m^{\text{UV}}) \cdot Y, & Y \equiv \hat{m}_0 / \hat{\Lambda}_{\text{IR}}^{-(1+\gamma_m)} \quad (m_p \ll \Lambda_{\text{IR}} \ll \Lambda). \end{cases} \quad (6)$$



c.f.  $f_\pi L$  in recent lattice results: [1] Fodor et.al. ('11), [2] Cheng et.al. ('14), [3] DeGrand ('11).

# Summary

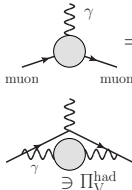
## 8-flavor QCD (LatKMI)

- Having Light  $\sigma$ , Showing Quasi-Conformal Nature with  $\gamma \sim 1$ .
- A viable candidate of Walking Technicolor Model (One-Family Model).

## SD with $\Lambda_{UV/IR}$ (Miura-Nagai-Shibata)

- The  $\gamma$  is strongly suppressed when the IR cutoff  $\Lambda_{IR}$  gets comparable to the scale  $(m_p, f_\pi)$ .
- The SD-based FSHS allows us to avoid the suppression, explains how two slopes in FSHS result from the IR cutoffs.
- The formulas applicable for lattice mass spectra are desirable.

# LABEX-OCEVU Project in CPT Aix-Marseille Univ: Muon ( $g - 2$ )



$$= -ie[\gamma_\nu F_1(q^2) + \frac{i\sigma^{\nu\rho}q_\rho}{2m_{\text{muon}}} F_2(q^2)]$$

$$a_\mu = (g - 2)/2 \rightarrow F_2(0)$$

$\ni \Pi_V^{\text{had}}$

$$S \equiv 4\pi\Pi'_{V-A}(Q^2 \rightarrow 0)$$



$$\Pi_{V-A} \sim \text{EW} \text{ (blob) EW}$$

There exists  $3.3\sigma$  deviation between  $a_\mu^{\text{exp}}$  and  $a_\mu^{\text{SM}}$ .