

Fermionic UV completions of partial compositeness: classification and phenomenology

(based on 1312.5330 with D. Karateev, and 1404.7137)



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PLAN

- ▶ *Brief* Introduction
- ▶ The idea of fermionic UV completions of partial compositeness
- ▶ Classification of models (and some caveats)
- ▶ Sketch of three models and their main properties:
 - $Sp(4)$ hypercolor [Barnard, Gherghetta, Ray: 1311.6562]
 - $SU(4)$ hypercolor [G.F.: 1404.7137, Golterman, Shamir: 1502.00390]
 - $SU(3)$ hypercolor [Vecchi: 1506.00623]
- ▶ Comments on light neutral scalars, ALPs, DM etc...
- ▶ Conclusions and wish list for the Lattice

The masses of the particles we consider elementary today span many orders of magnitudes. The Higgs mechanism does an excellent job at parameterizing the spectrum in a consistent way, but leaves many “qualitative” questions unanswered, such as:

1. Why is the Higgs mass itself so low?
2. How do we explain the huge disparity among fermion masses?

One possible explanation to 1. is to realize the Higgs as a (pseudo) Nambu-Goldstone Boson (pNGB) of a broken global symmetry.

[Georgi, Kaplan] (“Composite Higgs”).

One way to cope with the disparity of fermionic masses 2. without reintroducing fine-tuning is to also have additional “partners” to the SM fermions. [Kaplan] (“Partial Compositeness”).

Much work has been done in this area using the effective field theory description based on the CCWZ formalism. There was also a huge effort to realize these construction using extra-dimensions that I will not review.

Here we will look to an alternative proposal for constructing UV completions for these models. A proposal that is so old fashioned that it almost appears new. 😊

We will try to realize these models using four-dimensional gauge theories, based on some **hypercolor group** G_{HC} , with purely fermionic matter (**hyperfermions**). Fermionic models of BSM go all the way back to the old technicolor idea and were also tried in the context of **composite Higgs** and **partial compositeness**.

We will try to combine the two.

The goal is to start with the Higgsless and massless Standard Model

$$\mathcal{L}_{\text{SM0}} = -\frac{1}{4} \sum_{V=G,W,B} F_{\mu\nu}^2(V) + i \sum_{\psi=QudLe} \bar{\psi} \not{D} \psi$$

with gauge group $G_{\text{SM}} = SU(3) \times SU(2) \times U(1)$ and couple it to a theory $\mathcal{L}_{\text{comp.}}$ with hypercolor gauge group G_{HC} and global symmetry structure $G_F \rightarrow H_F$ such that

$$\mathcal{L}_{\text{comp.}} + \mathcal{L}_{\text{SM0}} + \mathcal{L}_{\text{int.}} \longrightarrow \mathcal{L}_{\text{SM}} + \cdots$$

$$\Lambda = 5 \sim 10 \text{ TeV}$$

($\mathcal{L}_{\text{SM}} + \cdots$ is the full SM plus possibly light extra matter from bound states of $\mathcal{L}_{\text{comp.}}$.)

Our goal is to find candidates for $\mathcal{L}_{\text{comp.}}$ and $\mathcal{L}_{\text{int.}}$ and to study their properties.

The interaction lagrangian $\mathcal{L}_{\text{int.}}$ typically contains a set of four-fermi interactions between hyperfermions and SM fermions, so the UV completion is only partial at this stage. However, we can imagine it being generated by integrating out d.o.f. from a theory \mathcal{L}_{UV} . (At a much higher scale because of e.g. flavor constraints.)

$$\begin{array}{ccc} \mathcal{L}_{\text{UV}} & \longrightarrow & \mathcal{L}_{\text{comp.}} + \mathcal{L}_{\text{SM0}} + \mathcal{L}_{\text{int.}} \longrightarrow \mathcal{L}_{\text{SM}} + \cdots \\ \Lambda_{\text{UV}} > 10^4 \text{ TeV} & & \Lambda = 5 \sim 10 \text{ TeV} \end{array}$$

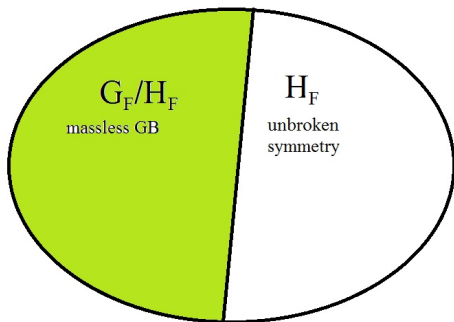
I will not attempt to construct such theory and will concentrate on the physics at the $5 \sim 10 \text{ TeV}$ scale, encoded in $\mathcal{L}_{\text{comp.}}$ and $\mathcal{L}_{\text{int.}}$.

We need to accomplish two separate tasks:

- ▶ Give mass to the vector bosons.
- ▶ Give a mass to the fermions. (In particular the top quark.)

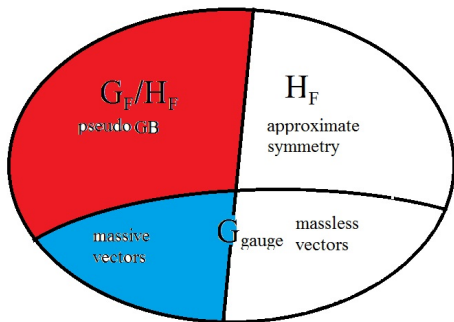
Let's start with the first one.

Let $\mathcal{L}_{\text{comp.}}$ have a global symmetry G_F spontaneously broken to H_F by a vacuum condensate $\langle\psi\psi\rangle$.



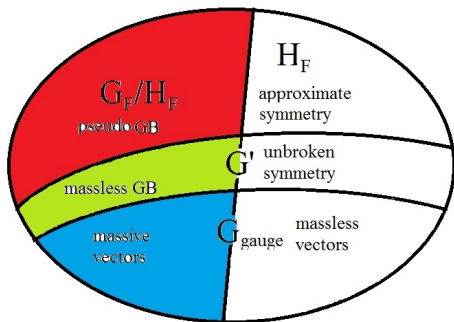
There will be a massless Goldstone boson for every broken generator G_F/H_F .

Consider gauging a *generic* (anomaly free) subgroup G_{gauge} of G_F .



Some Goldstone bosons are either eaten by the vectors, or become pseudo Goldstone bosons.

Let us mention that the general case is when the full theory preserves a global G' of which only $G_{\text{gauge}} \subset G'$ is gauged.



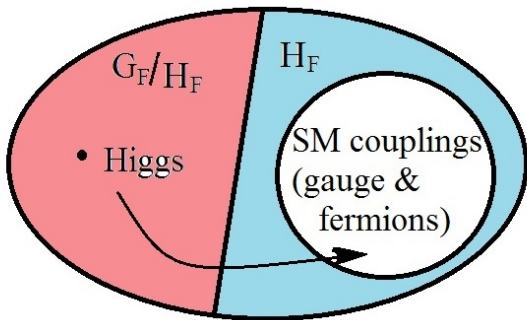
Some Goldstone boson are eaten by the vectors, some become massive pseudo Goldstone bosons and some remain massless.

This picture however is a bit misleading.

Looking at these pictures, one would think that if one took $G_{\text{gauge}} \subset H_F$ all gauge bosons would remain massless. But this is not necessarily true, since quantum corrections may **misalign** the vacuum, forcing some of the pNGB in G_F/H_F to condense and break G_{gauge} as a secondary effect. (There are some conditions that need to be met [Witten]. We will rely on the coupling to fermions as in e.g. [Agashe, Contino, Pomarol: 0412089].)

This leads to a possible resolution of the problem generically arising in technicolor theories where, breaking the SM group directly via the condensate typically leads to too large corrections to the S-parameter.

So the picture we have in mind is:



More specifically, to preserve custodial symmetry and to be able to give the correct hypercharge to all SM fields, we need

- ▶ $SU(2)_L \times SU(2)_R \times U(1)_X \subseteq H_F$
- ▶ $\text{Higgs} = (\mathbf{2}, \mathbf{2})_0 \in G_F/H_F$

The three “basic” cosets one can realize with fermionic matter

For a set of n irreps the hypercolor group:

| | |
|---|---|
| $(\psi_\alpha, \tilde{\psi}_\alpha)$ Complex | $\langle \tilde{\psi}\psi \rangle \neq 0 \Rightarrow SU(n) \times SU(n)' / SU(n)_D$ |
| ψ_α Pseudoreal | $\langle \psi\psi \rangle \neq 0 \Rightarrow SU(n) / Sp(n)$ |
| ψ_α Real | $\langle \psi\psi \rangle \neq 0 \Rightarrow SU(n) / SO(n)$ |

(The $U(1)$ factors need to be studied separately because of possible ABJ anomalies.)

The first case is just like ordinary QCD: $\langle \tilde{\psi}^{\alpha ai} \psi_{\alpha aj} \rangle \propto \delta_j^i$ breaks $SU(n) \times SU(n)' \rightarrow SU(n)_D$

In the other two cases, a **real/pseudo-real** irrep of the hypercolor group possesses a **symmetric/anti-symmetric** invariant tensor $t^{ab} = \delta^{ab} / \epsilon^{ab}$ making the condensate $t^{ab} \langle \psi_a^{\alpha i} \psi_{\alpha b}^j \rangle$ also **symmetric/anti-symmetric** in i and j , breaking $SU(n) \rightarrow SO(n)$ or $Sp(n)$.

As far as the EW sector is concerned, the possible minimal custodial cosets of this type are

| | |
|---|---------------------------------|
| $4 (\psi_\alpha, \tilde{\psi}_\alpha)$ Complex | $SU(4) \times SU(4)' / SU(4)_D$ |
| $4 \psi_\alpha$ Pseudoreal | $SU(4) / Sp(4)$ |
| $5 \psi_\alpha$ Real | $SU(5) / SO(5)$ |

E.g. $SU(4)/SO(4)$ is not acceptable since the pNGB are only in the symmetric irrep $(\mathbf{3}, \mathbf{3})$ of $SO(4) = SU(2)_L \times SU(2)_R$ and thus we do not get the Higgs irrep $(\mathbf{2}, \mathbf{2})$.

pNGB content under $SU(2)_L \times SU(2)_R$: ($X = 0$ everywhere)


- ▶ **Ad** of $SU(4)_D \rightarrow (\mathbf{3}, \mathbf{1}) + (\mathbf{1}, \mathbf{3}) + 2 \times (\mathbf{2}, \mathbf{2}) + (\mathbf{1}, \mathbf{1})$
- ▶ **A₂** of $Sp(4) \rightarrow (\mathbf{2}, \mathbf{2}) + (\mathbf{1}, \mathbf{1})$
- ▶ **S₂** of $SO(5) \rightarrow (\mathbf{3}, \mathbf{3}) + (\mathbf{2}, \mathbf{2}) + (\mathbf{1}, \mathbf{1})$

Some remarks

- ▶ Of course, since $SU(4) = SO(6)$ and $Sp(4) = SO(5)$ the cosets can be written in various ways.
- ▶ We will need to enlarge this construction to accommodate color.
- ▶ For these NGB we can take $Y = T_R^3$ but for the fermionic partners we will need an extra $U(1)_X$ to generate the right $Y = T_R^3 + X$.

Let's move on to the fermionic partners

The fermionic masses also present two options.

First try a bilinear term  (dropping all coupling constants and group indices, q = generic SM fermion, ψ = generic hyperfermion).

Starting at Λ_{UV} with terms like

$$\mathcal{L}_{\text{int.}} = \frac{1}{\Lambda_{\text{UV}}^2} \psi\psi qq + \frac{1}{\Lambda_{\text{UV}}^2} qqqq + \dots$$

Generically $\Lambda_{\text{UV}} > 10^7 \text{ GeV}$ to avoid FCNC terms in $\frac{1}{\Lambda_{\text{UV}}^2} qqqq$.

Going down to the confinement scale Λ , allowing for (large negative) anomalous dimension as in e.g. walking/conformal technicolor

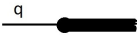
$$[\psi\psi]_{\Lambda_{\text{UV}}} = \left(\frac{\Lambda}{\Lambda_{\text{UV}}} \right)^\gamma [\psi\psi]_\Lambda$$

...yields, after condensation: (*very* schematically)

$$m_q \approx \Lambda \left(\frac{\Lambda}{\Lambda_{\text{UV}}} \right)^{2+\gamma}$$

To get the top quark mass we need $\left(\frac{\Lambda}{\Lambda_{\text{UV}}} \right)^{2+\gamma} \approx 1$ and this can happen:

- ▶ if $\Lambda \approx \Lambda_{\text{UV}}$. But this reintroduces the fine-tuning since Λ_{UV} generically must be very large to suppress unwanted $qqqq$ interactions.
- ▶ if $\gamma \approx -2$. But this means that $H = [\psi\psi]_\Lambda$ is almost a free field having dimension: $\Delta[H] \approx 3 - 2 = 1$ and thus $H^\dagger H$ becomes strongly relevant having scaling dimension: $\Delta[H^\dagger H] \approx 1 + 1 = 2 < 4$, reintroducing the fine-tuning of the Higgs bilinear.

The other way of doing it (**partial compositeness** [Kaplan]) is to have a mixing linear in q : $\frac{1}{\Lambda_{UV}^2} q \psi \psi \psi =$  and EWSB mediated by the strong sector:



$$\mathcal{L}_{\text{int.}} = \frac{1}{\Lambda_{UV}^2} \psi \psi \psi q + \frac{1}{\Lambda_{UV}^2} q q q q + \dots$$

Going down to the confinement scale Λ one now interpolates the *fermionic* field:

$$[\psi \psi \psi]_{\Lambda_{UV}} = \left(\frac{\Lambda}{\Lambda_{UV}} \right)^{\gamma'} [\psi \psi \psi]_{\Lambda}$$

... yielding, after condensation: (again, schematically dropping all couplings)

$$m_q \approx \Lambda \left(\frac{\Lambda}{\Lambda_{\text{UV}}} \right)^{2(2+\gamma')}$$

To get the right top quark mass we still need $\Lambda \approx \Lambda_{\text{UV}}$ or $\gamma' \approx -2$, but now the second option is **not fine tuned** because it refers to $T \propto [\psi\psi\psi]_\Lambda$ of classical dimension $9/2$.

Also notice that $\gamma' \approx -2$ is still **strictly above** the unitarity bound for fermions: ($\Delta[T] \approx 9/2 - 2 = 5/2 > 3/2$), contrary to $\gamma \approx -2$ for H which we saw is at the **free field limit**.

No new relevant operators are reintroduced in this case.

In many cases it is not possible to construct partners to all the SM fermions, so one could try a compromise: Use “partial compositeness” for the top sector and the usual bilinear term for the lighter fermions. [Matsedonskyi: 1411.4638, Cacciapaglia et al.: 1501.03818].

What is non negotiable in this approach is the existence of at least ψ^3 hypercolor singlets $\in (\mathbf{3}, \mathbf{2})_{1/6}$ and $(\mathbf{3}, \mathbf{1})_{2/3}$ of G_{SM} .
(The fermionic partners to the third family (t_L, b_L) and t_R .)

In the composite sector they arise as Dirac fermions and only one chirality couples to the SM fields.

If one had scalars in the theory $\mathcal{L}_{\text{comp}}$, one could make G_{HC} invariants of the right scaling dimension by taking simply $\psi\phi$, but of course, this reintroduces the naturalness issue.

If some fermions are in the Adjoint of G_{HC} , one has also the option $T = \psi\sigma^{\mu\nu}F_{\mu\nu}$ of naive dim. $\Delta[T] = 7/2$ requiring only $\gamma \approx -1$

Since we want to obtain the top partners, we also need to embed the color group $SU(3)_c$ into the global symmetry of $\mathcal{L}_{\text{comp.}}$. (This is not discussed much in the “CCWZ” literature.)

The minimal field content allowing an anomaly-free embedding of unbroken $SU(3)_c$ are

| | |
|--|--|
| $3 (\chi_\alpha, \tilde{\chi}_\alpha)$ Complex | $SU(3) \times SU(3)' \rightarrow SU(3)_D \equiv SU(3)_c$ |
| $6 \chi_\alpha$ Pseudoreal | $SU(6) \rightarrow Sp(6) \supset SU(3)_c$ |
| $6 \chi_\alpha$ Real | $SU(6) \rightarrow SO(6) \supset SU(3)_c$ |

In this case, we don't *need* to have a condensate and one could also use bare masses avoiding extra pNGBs.

The QCD quantum numbers of the χ s and their possible invariant mass terms are (writing only $SU(3)_c$ indices)

Complex case $(\chi^{1,2,3}, \tilde{\chi}_{1,2,3}) \in (\mathbf{3}, \bar{\mathbf{3}})$

Pseudoreal case $\chi^{1,2,3} \in \mathbf{3} \quad \chi^{4,5,6} \in \bar{\mathbf{3}}$

Real case $(\chi^1 + i\chi^4, \chi^2 + i\chi^5, \chi^3 + i\chi^6) \in \mathbf{3}$
 $(\chi^1 - i\chi^4, \chi^2 - i\chi^5, \chi^3 - i\chi^6) \in \bar{\mathbf{3}}$

The mass terms invariant under $SU(3)_D$, $Sp(6)$ and $SO(6)$ respectively are

| | |
|------------------------|--|
| Complex case | $m(\chi^1 \tilde{\chi}_1 + \chi^2 \tilde{\chi}_2 + \chi^3 \tilde{\chi}_3)$ |
| Pseudoreal case | $m(\chi^1 \chi^4 + \chi^2 \chi^5 + \chi^3 \chi^6)$ |
| Real case | $m(\chi^1 \chi^1 + \chi^2 \chi^2 + \chi^3 \chi^3 + \chi^4 \chi^4 + \chi^5 \chi^5 + \chi^6 \chi^6)$ |

A broad classification

We have seen what the basic requirements are for the embedding of the (custodial) EW group and the color group. We require:

- ▶ $G_F \rightarrow H_F \supset \overbrace{SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X}^{\text{custodial } G_{\text{cus.}}} \supset G_{\text{SM}}$
- ▶ The MAC should not break neither G_{HC} nor $G_{\text{cus.}}$.
- ▶ G_{SM} free of 't Hooft anomalies. (We need to gauge it.)
- ▶ $G_F/H_F \ni (\mathbf{1}, \mathbf{2}, \mathbf{2})_0$ of $G_{\text{cus.}}$. (The Higgs boson.)
- ▶ ψ^3 hypercolor singlets $\in (\mathbf{3}, \mathbf{2})_{1/6}$ and $(\mathbf{3}, \mathbf{1})_{2/3}$ of G_{SM} .
(The fermionic partners to the third family (t_L, b_L) and t_R .)
- ▶ B and L symmetry.

We shall restrict to the case where G_{HC} is simple and the fermion content is non chiral. (The last condition essentially follows from the others.)

Let us first consider the case where we have only one type of irrep. for the fermions.

★ Suppose we only have n L.H. fermions ψ belonging to the same real irrep. Then we expect the $SU(n)/SO(n)$ coset. Requiring that the whole $G_{\text{cus.}} = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X$ to be accommodated in $SO(n)$ gives $n \geq 10$, but actually we need $n \geq 11$ to have $H \in (1, 2, 2)_0$.

The only hypercolor groups that allow the existence of ψ^3 partners are G_2 and F_4 with the fundamental irrep. \mathbf{F} .

I don't believe these "GUT" cases are phenomenologically relevant given the issues with leptoquarks [Gripaios: 0910.1789] and proton decay. (For a different GUT coset, based on $SO(11)/SO(10)$, see [Frigerio, Serra, Varagnolo: 1103.2997].)

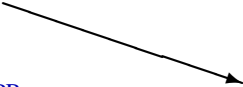
★ n fermions ψ belonging to the same **pseudoreal irrep.** would give rise to a $SU(n)/Sp(n)$ coset. But this will never work since it does not allow for ψ^3 hypercolor singlets.

★ n pairs of L.H. fermions $(\psi, \tilde{\psi})$ in **complex conj. irreps** would give rise to $SU(n) \times SU(n)' / SU(n)_D$ (times a vector-like $U(1)$ to be discussed later) just as in good old technicolor. In our original classification we dismissed this case also because the NGB arise as pseudo-scalars, like the pions. However [Vecchi: 1506.00623] has argued that this is acceptable.

He has also pointed out that for $n \geq 9$ **F** irreps and $G_{\text{HC}} = SU(3)$ it is possible to realize both the composite Higgs and partners to **all the SM fermions**. (To just meet the minimal conditions above it's enough to have $n = 7$. This construction also works for $G_{\text{HC}} = SU(6)$, with the **A₂** or $G_{\text{HC}} = E_6$, with the **F**.)

We can move on to consider the case where the EW physics and the QCD physics are controlled by two different irreps.

For instance, we could try to use 5 real irreps to get the EW coset $SU(5)/SO(5)$ and 3 complex pairs of irreps to get $SU(3) \times SU(3)'/SU(3)_c$. All other combinations of R, PR and C irreps are possible. (Together with D. Karateev, we classified these cases except for when complex irreps are used to generate the EW coset.)



| | R | PR | C |
|----|--|--|--|
| R | $\frac{SU(5)}{SO(5)} \frac{SU(6)}{SO(6)} U(1)$ | $\frac{SU(4)}{Sp(4)} \frac{SU(6)}{SO(6)} U(1)$ | $\frac{SU(4) \times SU(4)'}{SU(4)_D} \frac{SU(6)}{SO(6)} U(1)^2$ |
| PR | $\frac{SU(5)}{SO(5)} \frac{SU(6)}{Sp(6)} U(1)$ | $\frac{SU(4)}{Sp(4)} \frac{SU(6)}{Sp(6)} U(1)$ | $\frac{SU(4) \times SU(4)'}{SU(4)_D} \frac{SU(6)}{Sp(6)} U(1)^2$ |
| C | $\frac{SU(5)}{SO(5)} \frac{SU(3) \times SU(3)'}{SU(3)_D} U(1)^2$ | $\frac{SU(4)}{Sp(4)} \frac{SU(3) \times SU(3)'}{SU(3)_D} U(1)^2$ | $\frac{SU(4) \times SU(4)'}{SU(4)_D} \frac{SU(3) \times SU(3)'}{SU(3)_D} U(1)^3$ |

We can move on to consider the case where the EW physics and the QCD physics are controlled by two different irreps.

For instance, we could try to use **5 real irreps** to get the EW coset $SU(5)/SO(5)$ and **3 complex pairs of irreps** to get $SU(3) \times SU(3)' / SU(3)_c$. All other combinations of R, PR and C irreps are possible. (Together with D. Karateev, we classified these cases except for when complex irreps are used to generate the EW coset.)

| | R | PR | C |
|----|--|--|--|
| R | $\frac{SU(5)}{SO(5)} \frac{SU(6)}{SO(6)} U(1)$ | $\frac{SU(4)}{Sp(4)} \frac{SU(6)}{SO(6)} U(1)$ | $\frac{SU(4) \times SU(4)'}{SU(4)_D} \frac{SU(6)}{SO(6)} U(1)^2$ |
| PR | $\frac{SU(5)}{SO(5)} \frac{SU(6)}{Sp(6)} U(1)$ | $\frac{SU(4)}{Sp(4)} \frac{SU(6)}{Sp(6)} U(1)$ | $\frac{SU(4) \times SU(4)'}{SU(4)_D} \frac{SU(6)}{Sp(6)} U(1)^2$ |
| C | $\frac{SU(5)}{SO(5)} \frac{SU(3) \times SU(3)'}{SU(3)_D} U(1)^2$ | $\frac{SU(4)}{Sp(4)} \frac{SU(3) \times SU(3)'}{SU(3)_D} U(1)^2$ | $\frac{SU(4) \times SU(4)'}{SU(4)_D} \frac{SU(3) \times SU(3)'}{SU(3)_D} U(1)^3$ |

Three of them do not give fermionic partners.

For completeness, the full list of solutions is

$$\frac{G_F}{H_F} = \frac{SU(5)}{SO(5)} \frac{SU(6)}{SO(6)} U(1)$$

| G_{HC} | ψ | χ | Restrictions |
|---------------------|--------------------------|--------------------------|-------------------------|
| $SO(N_{\text{HC}})$ | $5 \times \mathbf{S}_2$ | $6 \times \mathbf{F}$ | $N_{\text{HC}} \geq 55$ |
| $SO(N_{\text{HC}})$ | $5 \times \mathbf{Ad}$ | $6 \times \mathbf{F}$ | $N_{\text{HC}} \geq 15$ |
| $SO(N_{\text{HC}})$ | $5 \times \mathbf{F}$ | $6 \times \mathbf{Spin}$ | $N_{\text{HC}} = 7, 9$ |
| $SO(N_{\text{HC}})$ | $5 \times \mathbf{Spin}$ | $6 \times \mathbf{F}$ | $N_{\text{HC}} = 7, 9$ |

$$\frac{G_F}{H_F} = \frac{SU(5)}{SO(5)} \frac{SU(6)}{Sp(6)} U(1)$$

| G_{HC} | ψ | χ | Restrictions |
|----------------------|-------------------------|--------------------------|--------------------------|
| $Sp(2N_{\text{HC}})$ | $5 \times \mathbf{Ad}$ | $6 \times \mathbf{F}$ | $2N_{\text{HC}} \geq 12$ |
| $Sp(2N_{\text{HC}})$ | $5 \times \mathbf{A}_2$ | $6 \times \mathbf{F}$ | $2N_{\text{HC}} \geq 4$ |
| $SO(N_{\text{HC}})$ | $5 \times \mathbf{F}$ | $6 \times \mathbf{Spin}$ | $N_{\text{HC}} = 11, 13$ |

$$\frac{G_F}{H_F} = \frac{SU(5)}{SO(5)} \frac{SU(3) \times SU(3)'}{SU(3)_D} U(1)^2$$

| G_{HC} | ψ | $(\chi, \tilde{\chi})$ | Restrictions |
|---------------------|-------------------------|---|-----------------------------------|
| $SU(N_{\text{HC}})$ | $5 \times \mathbf{A}_2$ | $3 \times (\mathbf{F}, \bar{\mathbf{F}})$ | $N_{\text{HC}} = 4 \quad (\star)$ |
| $SO(N_{\text{HC}})$ | $5 \times \mathbf{F}$ | $3 \times (\mathbf{Spin}, \bar{\mathbf{Spin}})$ | $N_{\text{HC}} = 10, 14$ |

$$\frac{G_F}{H_F} = \frac{SU(4)}{Sp(4)} \frac{SU(6)}{SO(6)} U(1)$$

| G_{HC} | ψ | χ | Restrictions |
|----------------------|--------------------------|-------------------------|---|
| $Sp(2N_{\text{HC}})$ | $4 \times \mathbf{F}$ | $6 \times \mathbf{A}_2$ | $2N_{\text{HC}} \leq 36 \quad (\star\star)$ |
| $SO(N_{\text{HC}})$ | $4 \times \mathbf{Spin}$ | $6 \times \mathbf{F}$ | $N_{\text{HC}} = 11, 13$ |

(\star) [G.F.] $(\star\star)$ [Barnard, Gherghetta, Ray]

$$\frac{G_F}{H_F} = \frac{SU(4) \times SU(4)'}{SU(4)_D} \frac{SU(6)}{SO(6)} U(1)^2$$

| G_{HC} | $(\psi, \tilde{\psi})$ | χ | Restrictions |
|---------------------|--|-------------------------|-------------------------|
| $SO(N_{\text{HC}})$ | $4 \times (\mathbf{Spin}, \overline{\mathbf{Spin}})$ | $6 \times \mathbf{F}$ | $N_{\text{HC}} = 10$ |
| $SU(N_{\text{HC}})$ | $4 \times (\mathbf{F}, \overline{\mathbf{F}})$ | $6 \times \mathbf{A}_2$ | $N_{\text{HC}} = 4$ (★) |

$$\frac{G_F}{H_F} = \frac{SU(4) \times SU(4)'}{SU(4)_D} \frac{SU(3) \times SU(3)'}{SU(3)_D} U(1)^3$$

| G_{HC} | $(\psi, \tilde{\psi})$ | $(\chi, \tilde{\chi})$ | Restrictions |
|---------------------|--|--|-----------------------------|
| $SU(N_{\text{HC}})$ | $4 \times (\mathbf{F}, \overline{\mathbf{F}})$ | $3 \times (\mathbf{A}_3, \overline{\mathbf{A}}_3)$ | $N_{\text{HC}} = 7$ |
| $SU(N_{\text{HC}})$ | $4 \times (\mathbf{F}, \overline{\mathbf{F}})$ | $3 \times (\mathbf{A}_2, \overline{\mathbf{A}}_2)$ | $N_{\text{HC}} \geq 5$ |
| $SU(N_{\text{HC}})$ | $4 \times (\mathbf{F}, \overline{\mathbf{F}})$ | $3 \times (\mathbf{S}_2, \overline{\mathbf{S}}_2)$ | $N_{\text{HC}} \geq 5$ |
| $SU(N_{\text{HC}})$ | $4 \times (\mathbf{A}_2, \overline{\mathbf{A}}_2)$ | $3 \times (\mathbf{F}, \overline{\mathbf{F}})$ | $N_{\text{HC}} \geq 5$ (★★) |
| $SU(N_{\text{HC}})$ | $4 \times (\mathbf{S}_2, \overline{\mathbf{S}}_2)$ | $3 \times (\mathbf{F}, \overline{\mathbf{F}})$ | $N_{\text{HC}} \geq 8$ |

(★) “switched model” (★★) “large N_{HC} model” [Golterman, Shamir]

THE $Sp(4)$ MODEL

| | $\overbrace{G_{\text{HC}}}$ | $\overbrace{G_{\text{F}}}$ | | |
|--------|-----------------------------|----------------------------|----------|---------|
| | $Sp(4)$ | $SU(4)$ | $SU(6)$ | $U(1)'$ |
| ψ | 4 | 4 | 1 | 3 |
| χ | 5 | 1 | 6 | -1 |

- ▶ The model is “non-chiral”, thus hypercolor group is free of gauge anomalies G_{HC}^3 .
- ▶ G_{F} is free of ABJ anomalies $G_{\text{F}}G_{\text{HC}}^2$.
- ▶ $H_{\text{F}} = Sp(4) \times SO(6)$ is free of 't Hooft anomalies H_{F}^3 .

QCD is embedded in the above $SO(6) \supset SU(3)_c \times U(1)_X$. Split the six χ into three pairs $(\chi \tilde{\chi}) \in (\mathbf{3}_{2/3}, \bar{\mathbf{3}}_{-2/3})$.

Bosonic mesons in the model:

| | $SU(4)$ | $SU(3)_c$ | $U(1)_X$ |
|---------------------------------|----------|--------------------|----------|
| $\psi^i \psi^j$ | 6 | 1 | 0 |
| $\chi^A \chi^B$ | 1 | 6 | 4/3 |
| $\tilde{\chi}_A \chi^B$ | 1 | 8 + 1 | 0 |
| $\tilde{\chi}_A \tilde{\chi}_B$ | 1 | $\bar{\mathbf{6}}$ | -4/3 |

After $\psi^i \psi^j$ condenses, $SU(4) \rightarrow Sp(4)$ and the **5** pNGB decompose as $(\mathbf{2}, \mathbf{2}) + (\mathbf{1}, \mathbf{1})$ of $SU(2)_L \times SU(2)_R$.

(**5** is the \mathbf{A}_2 of $Sp(4) \equiv \mathbf{F}$ of $SO(5)$).

This coset is discussed in [Gripaios, Pomarol, Riva, Serra: 0902.1483].

Fermionic composite operators in the model:

| | $SU(4)$ | $SU(3)_c$ | $U(1)_X$ |
|--|---------------|--------------------|----------|
| $\psi^i \chi^A \psi^j, \bar{\psi}_i \chi^A \bar{\psi}_j$ | 6 | 3 | $2/3$ |
| $\psi^i \tilde{\chi}_A \psi^j, \bar{\psi}_i \tilde{\chi}_A \bar{\psi}_j$ | 6 | $\bar{\mathbf{3}}$ | $-2/3$ |
| $\bar{\psi}_i \tilde{\chi}^A \psi^j$ | 15 + 1 | 3 | $2/3$ |
| $\bar{\psi}_i \bar{\chi}_A \psi^j$ | 15 + 1 | $\bar{\mathbf{3}}$ | $-2/3$ |

After SSB $SU(4) \rightarrow Sp(4)$, the low-energy fields decompose according to **6** \rightarrow **5** + **1** and **15** \rightarrow **10** + **5**.

THE $SU(4)$ MODEL

| | G_{HC} | | G_{F} | | | |
|----------------|-----------------------------|----------|----------------|-----------------------------|----------|---------|
| | $SU(4)$ | $SU(5)$ | $SU(3)$ | $SU(3)'$ | $U(1)_X$ | $U(1)'$ |
| ψ | 6 | 5 | 1 | 1 | 0 | -1 |
| χ | 4 | 1 | 3 | 1 | -1/3 | 5/3 |
| $\tilde{\chi}$ | $\bar{4}$ | 1 | 1 | $\bar{3}$ | 1/3 | 5/3 |

- ▶ The model is “non-chiral”, thus the hypercolor group is free of gauge anomalies G_{HC}^3 .
- ▶ G_{F} is free of ABJ anomalies $G_{\text{F}}G_{\text{HC}}^2$.
- ▶ $H_{\text{F}} = SO(5) \times SU(3)_c \times U(1)_X$ is free of 't Hooft anomalies H_{F}^3 .
Note that $G_{\text{cus.}} \subset H_{\text{F}} \subset G_{\text{F}}$.

There are various ways to argue that the symmetry breaking pattern should be $G_F \rightarrow H_F$ leading to the coset

$$G_F/H_F = \left(\frac{SU(5)}{SO(5)} \right) \times \left(\frac{SU(3) \times SU(3)'}{SU(3)_c} \right) \times U(1)'$$

Since this will be discussed in details by Maarten, I just give the decomposition:

$$\begin{array}{ccc} SO(5) \rightarrow SU(2)_L \times SU(2)_R \times U(1)_X & \rightarrow & SU(2)_L \times U(1)_Y \\ T_R^3 + X & = & Y \end{array}$$

The spectrum of light scalars thus comprises a Georgi-Machacek multiplet of the 14 NGB in $SU(5)/SO(5)$ (with $X = 0$) decomposing under $SO(5) \rightarrow SU(2)_L \times U(1)_Y$ as [Georgi, Machacek]

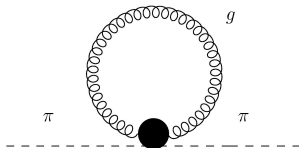
$$\mathbf{14} \rightarrow \mathbf{1}_0 + \mathbf{2}_{\pm 1/2} + \mathbf{3}_0 + \mathbf{3}_{\pm 1} \equiv (\eta, H, \Phi_0, \Phi_{\pm})$$

One more G_{SM} neutral boson η' arises from breaking $U(1)'$.

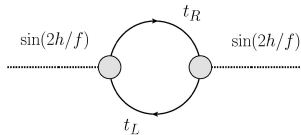
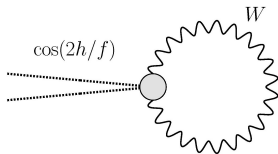
Finally there is a color octet Π^a arising from $SU(3) \times SU(3)' \rightarrow SU(3)_c$.

No leptoquarks or scalars in the **3** and **6** of QCD arise.

The colored pNGB octet gets a positive mass via the large contribution from gluons



whereas the coupling of the top quark favors the misalignment of the “right” Higgs boson. Some typical diagrams are (assuming factorization)



$$V(h) = \alpha \cos(2h/f) - \beta \sin^2(2h/f).$$

The top quark partners (for both $(t, b)_L$ and t_R) can be found as fermionic resonances created by the composite operators

| Object | $SO(5) \times SU(3)_c \times U(1)_X$ |
|--|---|
| $\tilde{\chi}\psi\tilde{\chi}, \quad \bar{\chi}\psi\bar{\chi}, \quad \bar{\chi}\bar{\psi}\tilde{\chi}$ | $(\mathbf{5}, \mathbf{3})_{2/3}$ |
| $\chi\psi\chi, \quad \tilde{\tilde{\chi}}\psi\tilde{\tilde{\chi}}, \quad \tilde{\tilde{\chi}}\bar{\psi}\chi$ | $(\mathbf{5}, \bar{\mathbf{3}})_{-2/3}$ |

The extra assumption we need to make is that at least one of these resonances is significantly lighter than the typical mass scale Λ . (This should be tested on the lattice.)

Again, Maarten will discuss this issue in more detail and I just give the decomposition:

After EWSB we end up with one Dirac fermion B of charge $-1/3$, three $T_{i=1,2,3}$ of charge $2/3$, and one X of charge $5/3$.

$$\begin{array}{ccc}
 SO(5) \times SU(3)_c \times U(1)_X & & (\mathbf{5}, \mathbf{3})_{2/3} \\
 \downarrow & & \downarrow \\
 G_{\text{cus.}} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X & & (\mathbf{3}, \mathbf{2}, \mathbf{2})_{2/3} + (\mathbf{3}, \mathbf{1}, \mathbf{1})_{2/3} \\
 \downarrow & & \downarrow \\
 G_{\text{SM}} \equiv SU(3)_c \times SU(2)_L \times U(1)_Y & & (\mathbf{3}, \mathbf{2})_{7/6} + (\mathbf{3}, \mathbf{2})_{1/6} + (\mathbf{3}, \mathbf{1})_{2/3} \\
 \downarrow & & \downarrow \\
 SU(3)_c \times U(1)_{\text{e.m.}} & & \mathbf{3}_{5/3} + 3 \times \mathbf{3}_{2/3} + \mathbf{3}_{-1/3}
 \end{array}$$

All the relevant couplings can be worked out by applying the CCWZ techniques.

The top quark has three partners and the full mass matrix turns out to be

$$\mathcal{M}_T = \begin{pmatrix} 0 & \frac{\lambda_q}{2}f(1 + \cos(v/f)) & \frac{\lambda_q}{2}f(1 - \cos(v/f)) & \frac{\lambda_q}{\sqrt{2}}f \sin(v/f) \\ \frac{\lambda_t}{\sqrt{2}}f \sin(v/f) & M & 0 & 0 \\ -\frac{\lambda_t}{\sqrt{2}}f \sin(v/f) & 0 & M & 0 \\ \lambda_t f \cos(v/f) & 0 & 0 & M \end{pmatrix}$$

whose lowest singular value is, to leading order in v/f , v/M

$$m_t \approx \frac{\sqrt{2}Mf\lambda_q\lambda_t}{\sqrt{M^2 + \lambda_q^2 f^2} \sqrt{M^2 + \lambda_t^2 f^2}} v,$$

The bottom quark has one partner B and a bilinear μ_b coupling is needed to give a mass to both.

The mass matrix is

$$\mathcal{M}_B = \begin{pmatrix} \mu_b \sin(v/f) \cos(v/f) & \lambda_q f \\ 0 & M \end{pmatrix}$$

The mass of the b quark is, to lowest order in the Higgs vev,

$$m_b \approx \frac{\mu_b M}{f \sqrt{M^2 + \lambda_q^2 f^2}} v$$

A positive feature of this model is that it does not give rise to large deviations from the $Z \rightarrow b\bar{b}$ decay rate.

This can be seen by noticing that the coupling of the B field to the Z boson turns out to be

$$\mathcal{L} \supset \frac{e}{s_w c_w} \left(-\frac{1}{2} + \frac{s_w^2}{3} \right) \bar{B} \gamma^\mu B Z_\mu$$

i.e. with the same coefficient as the SM b_L . This guarantees that no changes arise when rotating to the mass eigenbasis.

There are corrections to the (smaller) coupling to the b_R and to the t_L , t_R , but they are acceptable and might even be welcome.

This is of course the manifestation of the custodial symmetry [Agashe, Contino, Da Rold, Pomarol: 0605341] protecting $Z \rightarrow b_L b_L$ decay:

$$\text{Either } (T_L = T_R \text{ and } T_L^3 = T_R^3), \quad \text{or} \quad T_L^3 = T_R^3 = 0$$

In our case: $(T_L = T_R = \frac{1}{2} \text{ and } T_L^3 = T_R^3 = -\frac{1}{2})$

THE $SU(3)$ MODEL

| | G_{HC} | | G_{F} | |
|----------------|--------------------|----------|--------------------|--------|
| | $SU(3)$ | $SU(7)$ | $SU(7)$ | $U(1)$ |
| ψ | 3 | 7 | 1 | 1 |
| $\tilde{\psi}$ | $\bar{\mathbf{3}}$ | 1 | $\bar{\mathbf{7}}$ | -1 |

- ▶ Before coupling to the SM, the SSB is QCD-like
 $SU(7) \times SU(7)' \rightarrow SU(7)_D$
- ▶ So far, this is the same structure as technicolor but the embedding of G_{SM} is different.
- ▶ Embedding $G_{\text{SM}} \subset SU(7)_D$ allows to have fermionic partners of the type $\epsilon^{abc} \psi_a \psi'_b \psi''_c$. (G_{HC} indices.)
- ▶ By augmenting $7 \rightarrow 9$ one can actually find partners with the quantum numbers of all quarks.

Let us look at the minimal case.

Denote the 7 ψ by $\psi = (3 \times T, 2 \times D, S, S')$, (and similarly for the $\tilde{\psi}$).

| | G_{HC} | $G_{\text{SM}} \subset SU(7)$ | | |
|------|-----------------|-------------------------------|-----------|----------|
| | $SU(3)$ | $SU(3)_c$ | $SU(2)_L$ | $U(1)_Y$ |
| T | 3 | 3 | 1 | 0 |
| D | 3 | 1 | 2 | 1/3 |
| S | 3 | 1 | 1 | -1/6 |
| S' | 3 | 1 | 1 | 5/6 |

- ▶ “ TDS ” partner of Q_L , “ TDD ” or “ TSS' ” partners of u_R , “ TSS ” partners of d_R .
- ▶ $H_u \approx D\tilde{S}$, $H_d \approx D\tilde{S}'$.
- ▶ No proton decay in spite of QCD triplet scalars like $T\tilde{S}$, $T\tilde{S}'$ etc...
- ▶ Typical signature: collider stable hadrons with fractional charge due to a surviving vectorial $U(1)_T$.

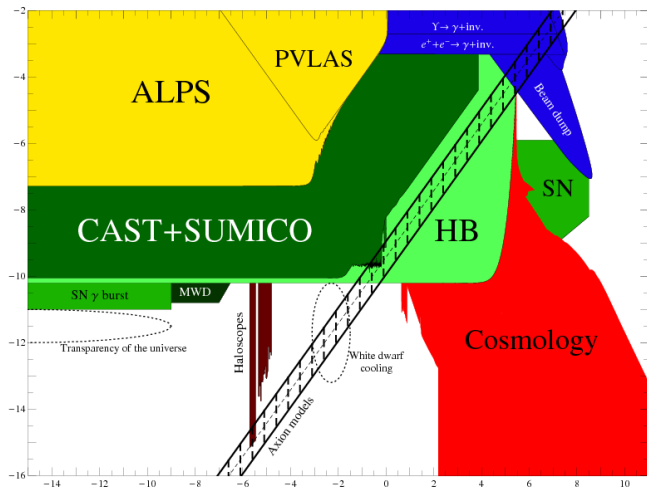
A closer look at the extra neutral scalars

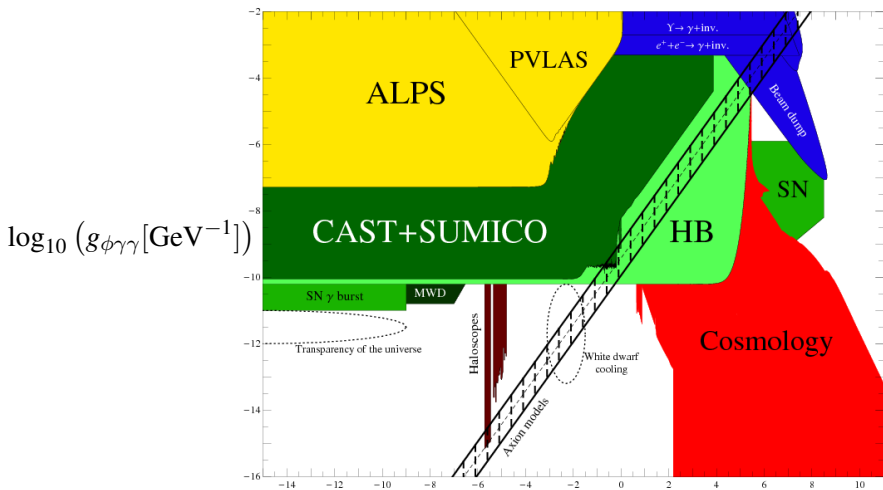
An unavoidable feature of these UV models is the presence of additional NGB neutral under the whole SM group and without CCWZ Yukawa couplings of the type discussed above.

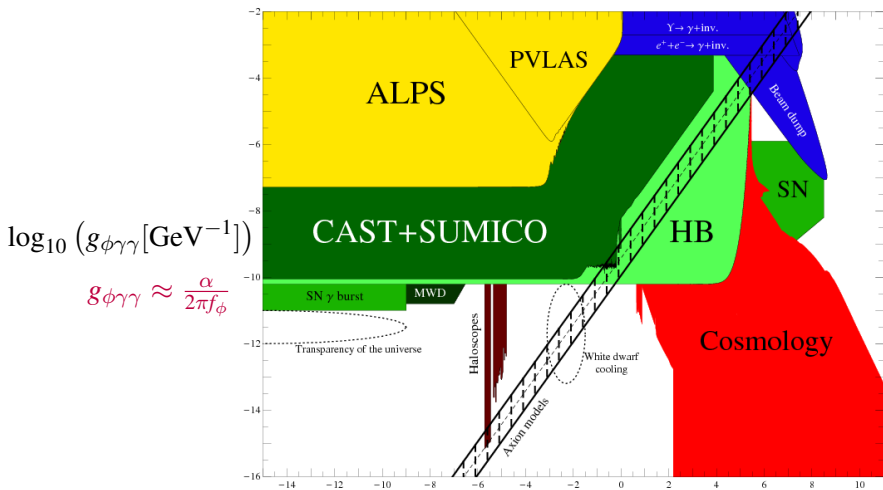
At this level, these NGB are massless, but this cannot be the whole story for obvious phenomenological reasons.

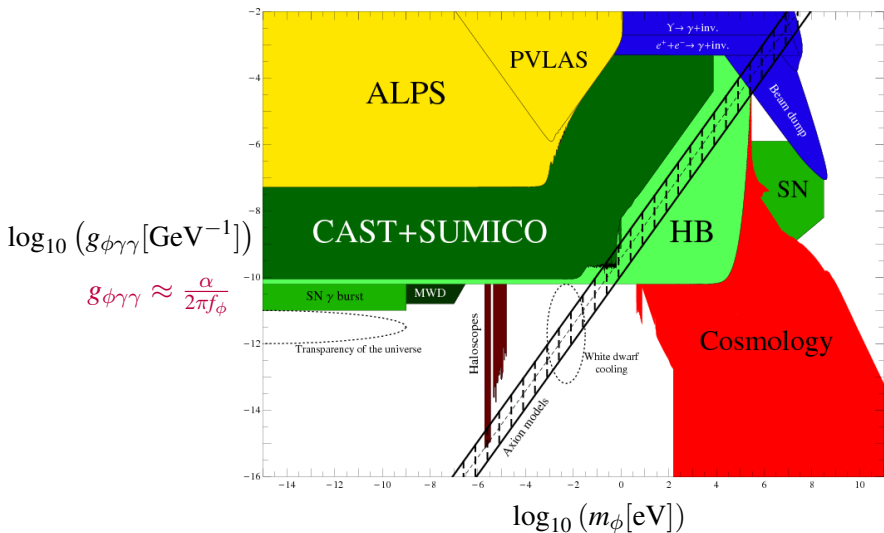
They can be given a mass via either bare masses for the hyperquarks or four-hyperquarks interactions, just as in technicolor. Some masses are also automatically generated by the anomalous couplings in analogy to 't Hooft's solution of the $U(1)_A$ problem.

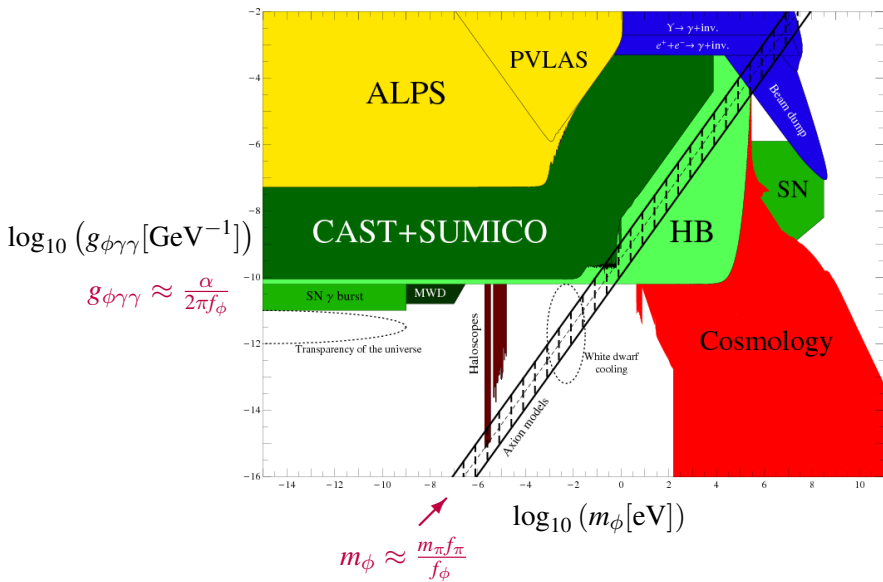
Let us start with a quick look at the current experimental situation:

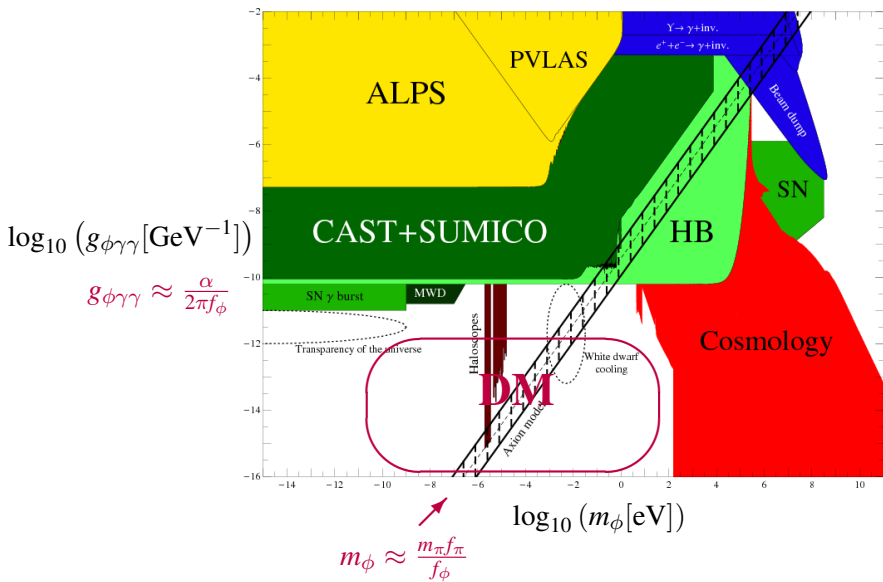


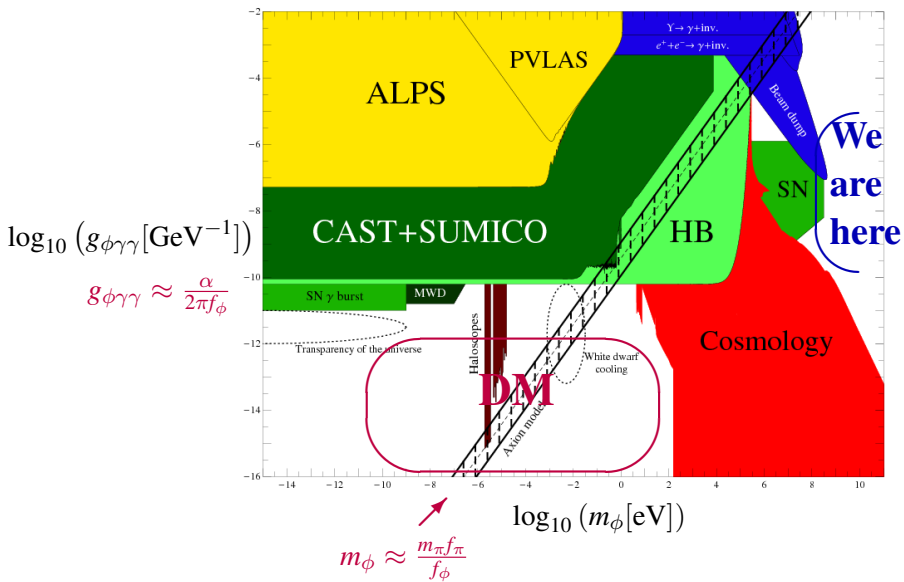


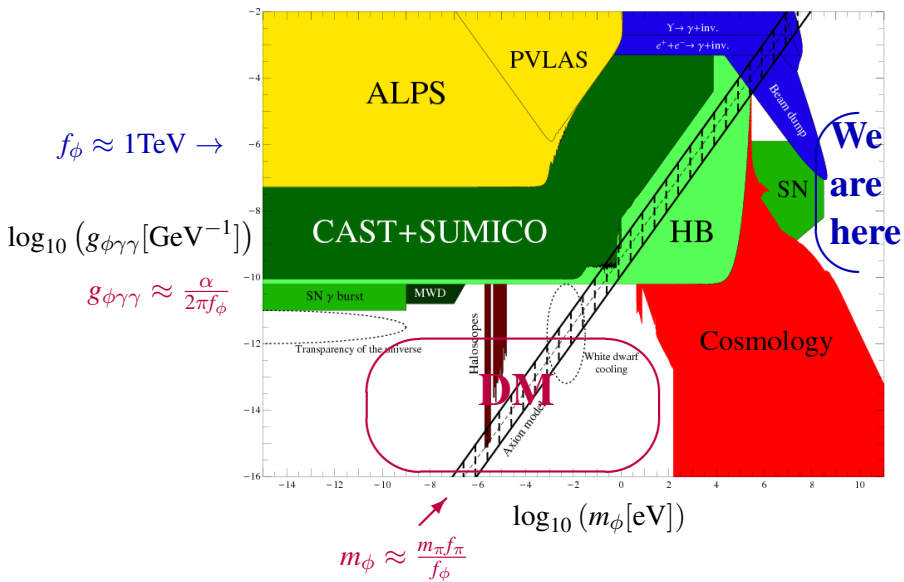


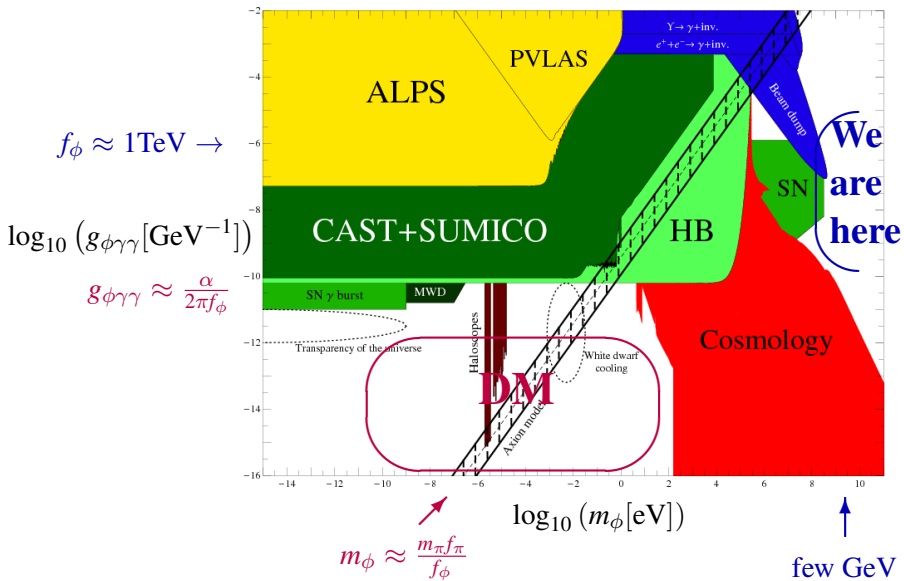












Consider the $G_{\text{HC}} = SU(4)$ model.

Before coupling it to anything it has three classical $U(1)$ symmetries.

| | $SU(4)$ | $U(1)_B$ | $U(1)_A$ | $U(1)_\psi$ |
|----------------|-----------------------------|----------|----------|-------------|
| ψ | 6 | 0 | 0 | 1 |
| χ | 4 | $-1/6$ | 1 | 0 |
| $\tilde{\chi}$ | $\bar{4}$ | $1/6$ | 1 | 0 |

$U(1)_A$ and $U(1)_\psi$ are broken by $\langle\psi\psi\rangle$, $\langle\chi\tilde{\chi}\rangle$. One linear combination is $SU(4)$ anomalous and does not give rise to a light scalar. The other combination gives rise to a version of **Kim's composite axion** after coupling to QCD. As it stands, $m \approx m_\pi f_\pi / f \approx 10$ keV excluded by constraints on stellar evolution.

Moreover, after coupling to the SM, one of the 14 NGB in $SU(5)/SO(5)$ remains massless (the η associated with $T_\eta = \text{diag}(1, 1, 1, 1, -4)$,). This symmetry is neither G_{HC} nor $SU(3)_c$ anomalous, however it does have a $U(1)_{\text{e.m.}}$ anomaly and so it is subjected to the same astrophysical constraints.

The only way to “save” models of this type is to give these particles a mass \gtrsim **few GeV** and this can be accomplished by bare masses or **(better?)** by the four-fermi terms arising at the Λ_{UV} scale.

This is also “old stuff”. In technicolor models, this was used to give a mass to the neutral axions also arising in these models [Farhi, Susskind].

$$H' = -\mathcal{L}_{4f} = \frac{1}{\Lambda_{UV}^2} (c_1 \chi^2 \tilde{\chi}^2 + c_2 \psi^4 + \dots)$$

For typical values of the parameters, using Dashen’s formula:

$$m^2 = \frac{1}{f^2} \langle [Q, [Q, H']] \rangle \approx \frac{\Lambda^6}{f^2 \Lambda_{UV}^2} \approx \frac{(5 \times 10^3 \text{ GeV})^6}{(800 \text{ GeV})^2 (10^8 \text{ GeV})^2} \approx (1.5 \text{ GeV})^2$$

but a fairly large range of masses is possible. Note however that Λ_{UV} cannot be arbitrarily large.

The same analysis can be carried out in general for all the other models [G.F., in progress].

Some differences do occur. For instance, in the $Sp(4)$ model, the second scalar, associated to the broken generator $\text{diag}(1, 1, -1, -1)$ of $SU(4)$, has neither a $SU(3)$ color nor a $U(1)$ e.m. anomaly, so it starts off at the lower left corner of the ALP plot.

If stable on a cosmological scale, some of these scalars could also provide DM candidates.

This is difficult for the models constructed so far. This is because the WZW term with SM gauging or the Higgs v.e.v. break the symmetry $\text{NGB} \rightarrow -\text{NGB}$ that could be used to keep them stable. The resulting decay rates are generically too large.

To avoid confusion, this comment does not apply to e.g. the SIMPs in [Hochberg, Kuflik, Volansky, Wacker: 1411.3727], where the WZW term is not gauged and the v.e.v.'s are zero.

In summary, my current understanding of the issue is the following: These models are acceptable insofar as the presence of neutral scalars is concerned, although these scalars don't yet help solving other issues such as DM or the strong CP problem.

This however could be just for lack of imagination. It is quite possible that there is an **extended theory** \mathcal{L}_{UV} giving rise to acceptable composite axions.

To have a proper composite axion, we need to address these issues far above the compositeness scale, given the constraints on the axion mass and the unavoidable relation $m \approx m_\pi f_\pi / f$.

Wish list for the lattice

For theories without Supersymmetry, the lattice is the only way of obtaining reliable quantitative predictions about the spectrum and other observables of confining gauge theories.

If BSM physics turns out to be described by one such theory, lattice will find a new crucial application beside the time-honored applications to QCD.

The first task, one that is already very much underway, is to understand the RG evolution and phase structure of BSM candidate theories. For instance, one would like to understand if a give models falls inside or outside of the conformal window [Work done by many of the members of the audience...].

By comparing with the closest available models simulated on the lattice, I believe that both the $Sp(4)$ and the $SU(4)$ models are outside the conformal window [Thanks to Pica and Sannino for discussions].

I should point out the following comparison between the two beta-functions:

$$\mu^2 \frac{d}{d\mu^2} \alpha = \beta(\alpha) = \begin{cases} -\frac{17}{12\pi} \alpha^2 + \frac{7}{16\pi^2} \alpha^3 + \dots & \text{for } Sp(4) \\ -\frac{7}{3\pi} \alpha^2 - \frac{461}{192\pi^2} \alpha^3 + \dots & \text{for } SU(4) \end{cases}$$

(In case you wonder, the zero for $Sp(4)$ is at $\alpha \approx 10$.)

As far as the $SU(3)$ model goes, more results are available, although the value of N_f where the window begins is still debated, as I understand. At any rate, it falls very near $N_f = 9$ which is the interesting case for the $SU(3)$ model.

Recent work on $SU(4)$ with A_2 irreps. and a discussion of their large- N_{HC} limit is found in [DeGrand et al. 1501.05665]. It could very well be that the large- N_{HC} limit of the full $SU(N_{HC})$ model enters the conformal window. (The beta-function turns around.)

More importantly, the viability of these models for BSM physics hinges upon the properties of the fermionic partners to the top quark, particularly if their masses are somewhat smaller than the confinement scale and if the associated composite operator acquires a large negative anomalous dimension.

I don't feel the requirement on the masses is too unrealistic. We need less that an order of magnitude suppression and the hyperquark content of these objects is such that they don't scale with N_{HC} like in QCD, they are more like "fermionic mesons". (See Maarten's talk.)

Hopefully the lattice community will think of simulations with multiple irreps. as a fun challenge more than a nuisance!

To wrap it up...

There is no denying the feeling we all have of standing at a crossroad.

The coming few years will tell us what awaits at the TeV scale and the status is unlikely to change until many of us retire...

There could be Supersymmetry 😊, Compositeness 😊, something we have not yet thought of 😊, or nothing 😱.

From the purely scientific point of view, each of these alternatives will be amazing, but the practical outcome for the community will be somewhat different...

The positive effect of this state of affairs is that it has brought closer many communities – experimentalists, lattice, model builders... and this nice workshop is a testimony to that.

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So, let's keep smiling and hope for the best!